

Mathematical Scepticism: the Cartesian Approach¹

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Introduction

Paris, 15 February 1665: Molière's *Don Juan* is first performed in the Palais-Royal Hall. Third Act, First Scene: the most daring of Don Juan's intellectual adventures takes place. In a dialogue with his servant Sganarelle, Don Juan makes explicit his atheist philosophy:

SGANARELLE. I want to get to the bottom of what you really think. Is it possible that you don't believe in Heaven at all?

D. JUAN. Let that question alone.

SGANARELLE. That means you don't. And Hell?

D. JUAN. Enough.

SGANARELLE. Ditto. What about the devil then?

D. JUAN. Oh, of course.

SGANARELLE. As little. Do you believe in an after life?

D. JUAN. Ha! ha! ha!

SGANARELLE. Here is a man I shall have a job to convert. [...]

SGANARELLE. But everybody must believe in something. What do you believe in?

D. JUAN. What do I believe?

SGANARELLE. Yes.

D. JUAN. I believe that two and two make four, Sganarelle, and four and four make eight.

SGANARELLE. That's a fine thing to believe! What fine article of faith! Your religion is then nothing but arithmetic. Some people do have queer ideas in their heads, and those that have been educated are often the silliest. I never studied, thank God, and no one can boast he taught me anything. But, to my poor way of thinking, my eyes are better than books. I know very well that this world we see around us is not a mushroom grown up in a single night. Who made those trees, those rocks, this earth, and that sky above there? Did all that make itself? There's yourself now. There you are. Did you come into being just like that? Didn't you father have to get your mother with child to make you? Can you see all the elements that go to make up the machine called man, without marvelling how they all fit in with one another? These nerves, these bones, these veins, these arteries, these... these lungs, this heart, this liver, and all the other ingredients there are, which... Oh, do please interrupt me. I can't go on arguing if you don't interrupt. You're holding your tongue on purpose, and letting me run on, just to make a fool of me..

D. JUAN. I was waiting until your argument was finished.

SGANARELLE. My argument is that there's something wonderful in Man, which none of your clever scientists can explain. I don't care what you say. Isn't it wonderful that I am standing here,

¹ This paper is part of ongoing research on mathematical scepticism. Some initial results have appeared in "Mathematical Scepticism: a Sketch with Historian in Foreground", in *The Skeptical Tradition around 1800*, edited by Johan van der Zande and Richard Popkin (Dordrecht: Kluwer, 1998), pp. 41-60. Two previous versions were given as invited papers at the session "The Pyrrhonian Problematic", *Twentieth World Conference* (Boston, 10-16 August, 1998) and at the *Journée de Paris XII — Val de Marne Sur le Scepticisme* (Paris, 23 October, 1998). I am grateful to those who took part in these two meetings, and especially to Georges Dicker and Jacques Brunschwig, for their questions and comments.

and that I have something in my head which makes me think a hundred different thoughts at once, and makes my body do whatever it likes? I clap my hands, raise my arms, look up to heaven, bow my head, move my feet, go to the right, go to the left, go forward, go back, turn round... [As he turns he falls down].

D. JUAN. Splendid! There lies your argument with a broken nose.

SGANARELLE. Oh, I'm a fool to waste my time arguing with you. Believe what you like. It's no affair of mine if you're damned.²

Had the elderly Descartes been in the audience, he would have found this dialogue rather disappointing. Obviously, Sganarelle had not read his *Meditations* (1641).³ Don Juan denies the existence of God and *believes* that $2 + 2 = 4$, but can he really be justified in maintaining that he *knows* that $2 + 2 = 4$ and hence transform arithmetic into the ultimate ground of certainty? Although Don Juan's internal doxastic state is obviously unassailable, in the *Replies* to the Second, Fifth, and Sixth set of *Objections* to the *Meditations*, Descartes had repeatedly argued that his epistemic position was far from being safe. Arithmetic cannot play the role of an indisputable religion because the mathematical atheist cannot justifiably claim to have absolutely certain knowledge even of simple mathematical truths. Either Don Juan accepts the ontological warrant provided by a benevolent God or he must give up all possible certainty, including that of mathematics. There are other difficulties as well. Descartes' *ante litteram* objection to Don Juan is original and controversial, but it is only the last in a series of sceptical arguments employed in the *Meditations* to test our confidence in the certainty of mathematical knowledge. Let us see what Sganarelle should have known.

The Background

The Project of the *Meditations* is to lay down new foundations for objective knowledge with absolute certainty and so make it possible to prove the existence of God and the immortality of the soul. The epistemological part of the project begins with a *pars destruens* in which the method of hyperbolic doubt is employed as a systematic and progressive means to test the whole set of beliefs that may, in theory, qualify as instances of scientific knowledge. In order to avoid confusion, we may refer to this discrete and extensional domain of scientific knowledge as the encyclopedia.⁴ The ultimate goal of the sceptical analysis is to discover whether there is any extension of the encyclopedia that is absolutely certain, and hence may serve as a ground on which to reconstruct (as a criterion to assess) other areas for further inclusion in the edifice

² Molière, *Don Juan and Other Plays*, tr. by G. Graveley and I. Maclean, ed. by I. Maclean (Oxford: Oxford University Press, 1989), pp. 61-62

³ In this article, the following standard abbreviations are used to refer to editions of Descartes: AT = C. Adam and P. Tannery (eds.), *Oeuvres de Descartes*, rev. ed., 12 vols. (Paris: Vrin-CNRS, 1964-76), references are by volume number (in roman) and page number (in Arabic). CSM = J. Cottingham, R. Stoothoff and D. Murdoch (eds.), *The Philosophical Writings of Descartes*, 2 vols. (Cambridge: Cambridge University Press, 1984), references by volume number (in roman) and page number (in Arabic). CSMK = J. Cottingham, R. Stoothoff and D. Murdoch A. Kenny (eds.), *The Philosophical Writings of Descartes*, vol. 3, *The Correspondence* (Cambridge: Cambridge University Press, 1991), references by volume number (in roman) and page number (in Arabic).

⁴ The "encyclopedia" that Descartes analyses is constituted by different types of scientific and objective information, and does not, nor should include, any psychologistic reference to Descartes' own linguistic, cognitive or logical capacities. The most obvious evidence of Descartes' extensionalist approach is provided not so much by the building metaphors that permeate the *Meditations*, as by the set-theoretic analysis of the collection of all his beliefs, which is twice compared to a basket of apples [AT VII, 481, 512; CSM II, 324, 349]. This explains why nowadays it is useful to compare beliefs to files stored in a digital memory. Note, however, that Descartes' *extensionalism*, though more consistent with his conceptual approach, is often obscured by the style of the *Meditations* and the philosophical subjectivism there advocated.

of scientific knowledge.

Clearly, the analysis of the encyclopedia cannot be complete without a full discussion of mathematics. Even nowadays, after the foundationalist crisis, mathematics is still the paradigm of the best knowledge available. Therefore, at the end of the First Meditation, sceptical doubts and mathematical certainties inevitably confront each other. The question is whether mathematics can provide the epistemological foundation of science.

Defending the absolute certainty of mathematical knowledge against scepticism represents the last chance to find within the encyclopedia itself the resources for its own validation. If mathematical certainties cannot be affected, even slightly, by sceptical doubts, then they can provide the ground on which we may assess, reconstruct and justify the rest of the system of knowledge. If even the mathematical area of the encyclopedia proves not to be immune from doubt and absolutely reliable, and fails to grant the ultimate justification of science, philosophical reflection itself is the only alternative means to provide the encyclopedia with a stable foundation, and the search for an ultimate criterion of certainty and truth needs to acquire a self-reflective nature, transforming the very process of investigation into a source of certainty.

Descartes argues that our faith in mathematical knowledge can indeed be undermined, and this is why, in the Second Meditation, the search for certainty moves on, acquires a self-reflective nature and reaches the *cogito*. The problem of the mathematical atheist occurs as an objection to this position. To see whether Descartes or Don Juan is right, we first need to outline the nature and force of the method of doubt with respect to mathematics.

The Method of Doubt

The method of doubt can be presented as a system of four interactive components:

- 1) the *constructivist requirement*: the endless number of beliefs (doxastic tokens), constituting the continuum of the encyclopedia, can be organised into a finite number of discrete classes (doxastic types) which can then be tested in a finite number of steps (AT VII, 18; CSM II, 12).
- 2) the *testing procedure*: a battery of sceptical arguments, working as epistemological *benchmarks*,⁵ is dialectically associated in a one-to-one relation with classes of beliefs (AT VII, 18; CSM II, 12).
- 3) the *twofold principle of maximum safety*: (i) no belief should be accepted as certain, and hence true, if it is ever possible to give any reason, no matter how questionable, unjustified, unlikely or implausible, for thinking that it might be false and hence for doubting its certainty (AT VII, 18; CSM II, 12); (ii) is then further reinforced by (i): a class of beliefs containing a member that can be doubted is to be treated as a class of doubtful beliefs and unconditional assent is to be withdrawn from the whole collection (AT VII, 18; CSM II, 12). Treating beliefs as files, the principle of maximum safety suggests that, if there is any suspicion that a file may have been corrupted by the sceptical virus, then not only it is better to avoid using that specific file, but it is also better to avoid using any other file with the same extension.
- 4) the *minimalist tactics*: the method aims at giving or withholding warranted assent to, not proving or disproving, (members of) classes of beliefs (AT VII, 18; CSM II, 12).

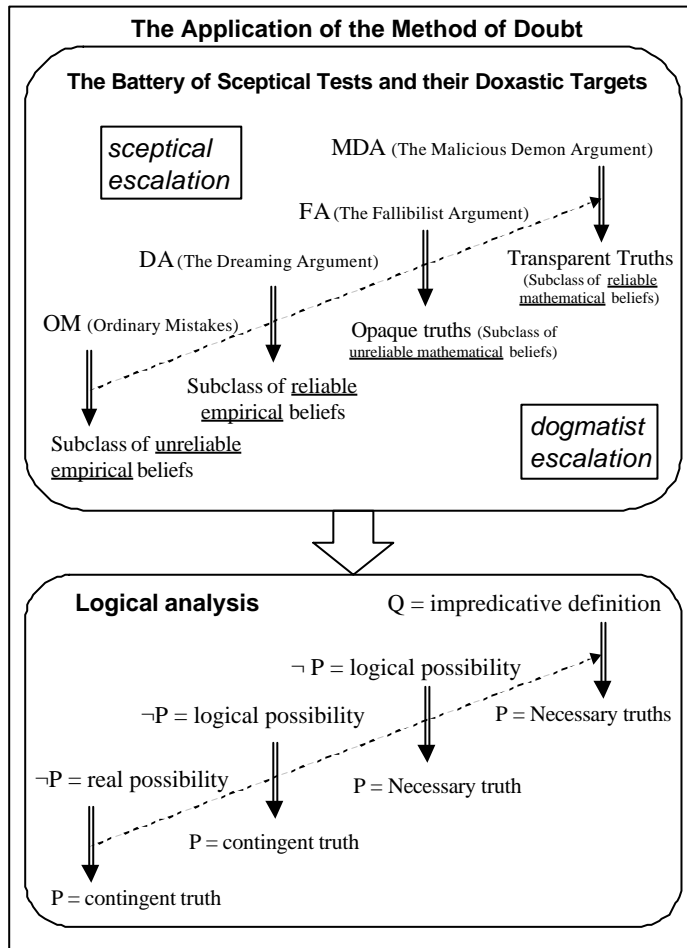
The outcome of the application of the method is that whole classes of doubtful beliefs (i.e. those possibly corrupted by the sceptical virus) are cognitively bracketed and placed, as

⁵ In information technology, a benchmark is a standard program or set of programs that can be run on different computers to provide a more or less accurate measure of their performance. A benchmark may evaluate the overall power of a system by including a standard selection of programs or it may attempt to measure more specific aspects of performance, like graphics, I/O, computation, specific tasks like rendering polygons, reading and writing files or performing operations on matrices. While no one benchmark can fully characterise overall system performance, the results of a variety of realistic benchmarks can give valuable insight into expected real performance.

temporary files, in a separate directory, in the hope that they will not all have to be erased, but that at least some may be rescued, once a safe criterion has been found whereby the degree of their epistemic reliability can be positively ascertained. The method is very radical, but the “degree to which it is radical” can be justified by the level of safety required by the foundationalist process, which in turn is consistent with the degree of importance of the project’s goal. The more important the epistemological end is, the greater the justification for extreme logical means.

The Sceptical Escalation

On the basis of the constructivist requirement, Descartes organises the encyclopedia into two doxastic classes, one empirical and the other mathematical. Both are then further divided into two sub-classes, reliable and unreliable, and each sub-class is examined through the testing procedure. Here is a schematic presentation:



The four classes have an increasing degree of epistemic reliability and hence require ever more powerful and subtle benchmarks to be tested.

The subclass of unreliable empirical beliefs is first challenged by the mere occurrence or actual possibility of *ordinary mistakes* (OM) and is then discharged, through the application of the principle of maximum safety, for failing to qualify as absolutely certain. The same holds true for the subclass of reliable empirical beliefs, which overcomes the problem of (OM) but then fails to pass the test of the dreaming argument (DA). However, after having applied (OM) and (DA), Descartes acknowledges that

whether I am awake or asleep, two and three added together are five, and a square has no more than four sides. It seems impossible that such *transparent truths* [*perspicuae veritates* my italics; Descartes uses the equivalent adverb *perspicue* in the Third Meditation, when speaking about mathematical truths *perspicue* intuited by the mind and the Malicious Demon Argument, see quotation below] should incur any suspicion of being false [AT VII, 20; CSM II, 14].

Later, in the Fifth Meditation, he recalls that:

I also remember that even before, when I was completely preoccupied with the objects of the sense, I always held that the most certain truths of all were the kind which I recognised clearly in connection with shapes, or numbers or other items relating to arithmetic or geometry, or in general to pure and abstract mathematics. [AT VII, 65; CSM II, 45]

In the escalation of “dogmatic”⁶ beliefs and sceptical doubts, the logical possibility of (DA) can override the contingent truth of any type of empirical knowledge, but is still defeated by the logical necessity of mathematical knowledge, though in a sense that will have to be carefully clarified in a moment. From the modal defeasability of (DA), it follows that the physical sciences that deal with the contingently true nature of the physical world, such as

physics, astronomy, medicine and all other disciplines which depend on the study of composite things, are doubtful; while arithmetic, geometry, and other subjects of this kind, which deal with the simplest and most general things, regardless of whether they really exist in nature or not, contain something certain and indubitable. [AT VII, 21; CSM II, 14]

Yet, according to Descartes, even mathematical truths can be shown not to be absolutely reliable beyond any possible doubt. To support this view, Descartes offers two arguments and a corollary.

The first argument, which I shall call the Fallibilist Argument (FA), is rather brief. Descartes discusses it and its corollary very succinctly in the First and Fifth Meditation (in the *Method*, it is treated before (DA) and together with (OM)). Following the logical order in which classes of beliefs are organised, (FA) has a propaedeutic role with respect to the second argument, the well-known Malicious Demon Argument (MDA), which is much more fully elaborated because more crucial.

The Fallibilist Argument

(FA) presupposes the following distinction, which is worth introducing before we look at the actual formulation of the argument.

According to Descartes, mathematical truths can be of two types:

OT) *opaque truths*—e.g. the compound interest of a bank account calculated on a yearly basis—are reached through, or consist of, mathematical practices, reasonings or techniques and may require the use of, or contain, other mathematical truths.

TT) *transparent truths* (*perspicuae veritates*, see the quotation above, AT VII, 20; CSM II, 13)—e.g. $2 + 3 = 5$ or “a square has no more than four sides”, but also the *cogito* and other

⁶ Here I am using the term “dogmatic” in the Pyrrhonian, technical sense, to refer to someone who is not a sceptic but positively asserts and defends philosophical theses as true or at least plausible.

“common notions”, i.e. innate principles and fundamental logical axioms—are self-evident, require no mathematical computation or logical elaboration but can be fully grasped intuitively and immediately, as clear and distinct truths, as long as the language in which they are expressed is known:

We cannot think of them without believing them to be true. The fact that I exist so long as I am thinking, or that what is done cannot be undone, are examples of truths in respect of which we manifestly possess this kind of certainty. For we cannot doubt them unless we think of them; but we cannot think of them without at the same time believing they are true, as was supposed. Hence we cannot doubt them without at the same time believing they are true; that is, we can never doubt them. [AT VII, 145-6; CSM II, 104]

At the end of his search, Descartes concludes that

But I know now that I am incapable of error in those cases where my understanding is transparently clear [At jam scio me in iis, quae *perspicue* [my italics] intelligo, falli non posse, AT VII, 71; CSM II, 49]

but, in the first part of the *Meditations*, the epistemic generation of an opaque truth is considered sufficiently complex to allow not only the logical but also the actual possibility of error (see the “imaginable” clause in the quotation below, AT VII, 21; CSM II, 14)—owing to human fallibility—and hence some sceptical doubts about the correctness of the mathematical result.

(FA) is successful only against (OT). The certainty of transparent truths is immediately obvious and can be appreciated by a single mental act. This is why the dogmatic answer to (DA) rightly relies on (TT): a transparent mathematical truth remains indubitable, and hence absolutely reliable for the foundationalist project, even if one is dreaming, so long as one attends to it.

Mathematical truths, nevertheless, do not all enjoy the same degree of epistemic indubitability. On the contrary, we do not have to be dreaming to make mistakes; we go astray so often even in such simple matters when we are awake, that no mathematical truth reached by a mediating process can satisfy the twofold principle of maximum safety. It is always logically possible to be wrong even in the most elementary calculation or logical reasoning. After all, we do often double-check even the simplest of arithmetical operations. Thus, human fallibility, in connection with the maximum safety principle, constitutes the sceptical benchmark for the first subclass of unreliable mathematical truths. We should withhold our unreserved and unconditional assent from the whole class of opaque mathematical truths, even from those depending on elementary processes of elaboration: (OT) cannot provide an absolutely certain foundation for the encyclopedia. Of course, this holds true also insofar as I do not immediately perceive that $2 + 3 = 5$ or “a square has no more than four sides” but actually “I add two and three or count the sides of a square”, as I did when I was a child.

If we turn now to the actual formulation of the Fallibilist Argument, we find it embedded in the following four passages, one from the *Discourse on the Method*, two from the *Meditations*, and the last from the *Principles of Philosophy*:

[FA.1] And since there are men who make mistakes in reasoning, committing logical fallacies concerning the simplest questions in geometry, and because I judged that I was prone to error as anyone else, I rejected as unsound all the arguments I had previously taken as demonstrative proofs. [AT VI, 32; CSM I, 127]

[FA.2] What is more, since I sometimes believe that others go astray in cases where they think they have the most perfect knowledge, may I not similarly go wrong every time I add two and three or count the sides of a square, or in some even simpler matters, if that is imaginable? [AT VII, 21; CSM II, 14]

[FA.3] Admittedly my nature is such that so long as [as soon as (French version)] I perceive something very clearly and distinctly I cannot but believe it to be true. By my nature is also such

that I cannot fix my mental vision continually on the same thing, so as to keep perceiving it clearly; and often the memory of a previously made judgement may come back, when I am no longer attending to the arguments which led me to make it. And so other arguments can now occur to me which might easily undermine my opinion, if I were unaware of God: and I should thus never have true and certain knowledge about anything, but only shifting and changeable opinions. For example, when I consider the nature of a triangle, it appears most evident to me, steeped as I am in the principles of geometry, that its three angles are equal to two right angles; and so long as I attend the proof, I cannot but believe this to be true. But as soon as I turn my mind's eye away from the proof, then in spite of still remembering that I perceived it very clearly, I can easily fall into doubt about its truth, if I am unaware of God. For I can convince myself that [*this is the step towards the malicious demon argument; note the temporal qualification, which shows that Descartes is still moving from the possibility of occasional mistakes—FA—to the possibility of constant mistakes—MDA*] I have a natural disposition to go wrong from time to time in matters which I think I perceive as evidently as can be. This will seem even more likely [*note the connection between the two arguments*] when I remember that [*this is the fallibilist argument*] there have been frequent cases where I have regarded things as true and certain, but have later been led by other arguments to judge them to be false. [AT VII, 69-70; CSM II, 48]

[FA.4, my emphasis] Our doubts will also apply to other matters which we previously regarded as most certain—even the demonstrations of mathematics and even the principles which we hitherto considered to be self-evident. *One reason* for this is that [*this is the fallibilist argument*] we have sometimes seen people make mistakes in such matters and accept a most certain and self-evident things which seemed false to us. *Secondly*, and most importantly, we have been told that [*this is the malicious demon argument*] there is an omnipotent God who created us. Now we do not know whether he may have wished to make us beings of the sort who are *always* [*my italics; contrast this temporal qualification with the previous version*] deceived even in those matters which seem to us supremely evident: for likely [*note the connection between the two arguments*] such *constant* [*my italics*] deception seems no less a possibility than the *occasional* [*my italics*] deception which, as we have noticed in previous occasions, does occur. We may of course suppose that our existence derives not from a supremely powerful God but either from ourselves or from some other source; but in that case, the less powerful we make the author of our coming into being, the more likely it will be that we are so imperfect as to be deceived all the time. [AT IXB, 6; CSM I, 194] [...] The mind, then, knowing itself, but still in doubt about all other things, looks around in all directions in order to extend its knowledge further. First of all, it finds within itself ideas of many things; and so long as it merely contemplates these ideas and does not affirm or deny the existence outside itself of anything resembling them it cannot be mistaken. Next, it finds certain common notions from which it constructs various proofs; and, for as long as it attends to them, it is completely convinced of their truth. For example, the mind has within itself ideas of numbers and shapes, and it also has such common notions such as: *If you add equals to equals the result will be equal*; from these it is easy to demonstrate that the three angles of a triangle equal two right angles, and so on. And so the mind will be convinced of the truth of this and similar conclusions, so long as it attends to the premises from which it deduced them. But it cannot attend to them all the time; and subsequently, [when it happens that it remembers a conclusion without attending to the sequence which enables it to be demonstrated (French version)] recalling that it is still ignorant as to whether it may have been created with the kind of nature that makes it go wrong even in matters which appear most evident, the mind sees that it has just cause to doubt such conclusions, and that the possession of certain knowledge will not be possible until it has come to know the author of its being. [AT IXB, 9-10; CSM I, 197]

(FA.1) and (FA.2) are almost identical: they make explicit the fact that (FA) is about the epistemic *genesis* of mathematical truths. (FA.3) further clarifies that even complex mathematical truths, such as the geometrical theorem about the internal angles of a triangle, can become *transparent* and yet, when treated as the results of mathematical processes, they still incur the same problem faced by *opaque* mathematical truths. The numbering of the two arguments in (FA.4) leaves no doubt about Descartes' intention to distinguish between (FA)

and the Malicious Demon Argument (MDA).⁷ In the last part of (FA.3) and in the first part of (FA.4), Descartes argues that, by reminding us of our *occasional* deception when dealing with some opaque mathematical truths, (FA) paves the way for MDA and the possibility that our deception may be *constant*. The connection between the two arguments is provided by the kind of nature that God may have given us: we may be beings of the sort who are occasionally (FA) or always (MDA) “deceived even in those matters which seem to us supremely evident”. The second part of the formulation in (FA.4) includes the argument against the atheist on which I shall concentrate later. If there is no God, then nature or mere chance are the sources of ourselves, but then, according to Descartes, “the less powerful we make the author of our coming into being, the more likely it will be that we are so imperfect as to be deceived all the time”.

Interestingly, the authors of the *Objections* do not contest the value of (FA). Mathematical fallibilism is implicitly accepted as unproblematic whenever processes and reasonings are in question. The only author who seems to discuss (FA), by way of an aside, is Bourdin, and his position only reinforces the Cartesian argument.

In formulating (FA), Descartes is addressing what, to us, looks like a Wittgensteinian problem of scientific practice and “following a rule”. This becomes completely clear in his reply to Bourdin:

For everyone knows that a distrustful person, as long as he remains in a state of distrust, and therefore does not affirm or deny anything, cannot be led into error even by an evil demon. But a man who adds two and three together can be deceived by such demon, as is shown by the example that my critic himself has produced concerning the man who counted one o'clock four times. [AT VII, 476; CSM II, 320]

Bourdin's paradoxical example mentioned by Descartes has a typically Wittgensteinian ring:

I know a man who once, when falling asleep, heard the clock strike four, and counted the strokes as “one, one, one, one”. It seemed to him that there was something absurd about this, and he shouted out: “That clock must be going mad; it has struck one o'clock four times!”. Is there really anything so absurd or irrational that it could not come into the mind of someone who is asleep or raving? There are no limits to what a dreamer may not prove or believe, and indeed congratulate himself on, as if he had managed to invent some splendid thought. [AT VII, 457; CSM II, 306]

By citing the paradoxical example of the clock, Bourdin means to argue that

- a) if hyperbolic doubt is taken seriously, then nothing at all can be certain for a person who is unsure whether he is asleep or not, not even that two plus three make five (AT VII, 457; CSM II, 306-7)
- b) Descartes' position, namely “if something appears certain to someone who is in doubt whether he is dreaming or awake, then it *is* certain—indeed so certain that it can be laid down as a basic principle of a scientific and metaphysical system of the highest certainty and exactness” is not as certain as the proposition that two plus three make five (AT VII, 457, see also 471, where the same point is restated, CSM II, 306-7, and 317).
- c) one can always have doubts or be deceived about the position presented in (b) by a malicious demon (AT VII, 457; CSM 306-7).

By arguing in favour of (a), Bourdin fails to grasp the distinction between transparent and opaque mathematical truths. Bourdin's example only reinforces Descartes' position, as Descartes himself makes clear in the quotation given above. By arguing in favour of (b), Bourdin confuses the conditional statement of a methodological criterion—P is certain if and only if P is absolutely indubitable, irrespective of the context in which P is formulated—with an extension of the encyclopedia. By arguing in favour of (c), Bourdin misunderstands the Cartesian strategy, and ends by reinforcing it: the point is not that the methodological criterion

⁷ Despite this clear evidence, the two arguments are sometimes treated as one, see for example Émile Bréhier “The Creation of the Eternal Truths in Descartes' System” (1937) repr. in W. Doney (ed.), *Descartes: a Collection of Critical Essays* (London: Macmillan, 1967), pp. 192-208, p. 200.

formulated in (b) is indubitable, but that if by means of adopting it, even if it is unreliable, we are able to reach an ultimate ground for certainty, then we have indeed reached certainty. As Descartes puts it, bad doubts are still doubts, and we may add that a poor methodology is still a methodology: if it yields the result, then any reservation about the latter immediately becomes pointless (note that this applies also to all objections concerning Descartes' more or less justified use of logic and language in the course of the *Meditations*).

The Malicious Demon Argument

The distinction *opaque* vs. *transparent* is strictly related to the distinction *simple* vs. *complex*, but it is not merely equivalent to it. All complex mathematical truths can be said to be opaque to the mind, but not all simple mathematical truths are necessarily transparent. If they are reached through some process then the latter's unreliability affects the epistemological status of the result, no matter how simple it is. Not all mathematical truths, however, are necessarily opaque either. By

running through a chain of inferences several times, simultaneously intuiting one relation on to the next, until I have learnt to pass from the first to last so swiftly that memory is left with practically no role to play [*Regulae*, Rule VII, AT X 388; CSM I, 25]

an opaque mathematical truth can become sufficiently simple to be fully intelligible through a single mental act. Against these simple mathematical truths *transparently* (*perspicue*) intuited, any claim of human fallibility appears to be out of the question. As Descartes suggests, it is imaginable that I may make a mistake even when adding two and three, but it is inconceivable that, *in re*, two and three added together may be either more or less than five:

But what about when I was considering something very simple and straightforward in arithmetic or geometry, for example that two and three added together make five, and so on? [The Latin text is more detailed, note the use of *perspicue*: Quid vero? Cùm circa res Arithmeticas vel Geometricas aliquid valde simplex et facile considerabam, ut quod duo et tria simul juncta sint quinque, vel similia, nunquid saltem illa satis *perspicue* intuebar, ut vera esse affirmarem?] Did I not see at least these things clearly enough to affirm their truth? Indeed, the only reason for my later judgement that they were open to doubt was that it occurred to me that perhaps some God could have given me a nature such that I was deceived even in matters which seemed most evident. And whenever [note that the Latin *Sed quoties..* is actually the first part of an adversative clause, and is connected with the following *Quoties vero...*; Haldane's translation is more accurate: "But as often as...; and, on the other hand, as often as..."] my preconceived belief in the supreme power of God comes to mind, I cannot but admit that it would be easy for him, if he so desired, to bring it about that I go wrong even in those matters which I think I see utterly clearly with my mind's eye. Yet *when* [*Quoties vero*] *I turn to the things themselves* [*ad ipsas res*, my emphasis] which I think I perceive very clearly, I am so convinced by them that I spontaneously declare: let whoever can do so deceive me, he will never bring it about that I am nothing, so long as I continue to think I am something; or make it true at some future time that I have never existed, since it is now true that I exist; or bring it about that two and three added together are more or less than five, or anything of this kind in which I see a manifest contradiction. And since I have no cause to think that there is a deceiving God, and I do not yet even know for sure whether there is a God at all, any reason for doubt which depends simply on this supposition is a very slight and, so to speak, metaphysical one. But in order to remove even this slight reason for doubt, as soon as the opportunity arises I must examine whether there is a God, and, if there is, whether he can be a deceiver. For if I do not know this, it seems that I can never be quite certain about anything else. [AT VII, 36; CSM II, 25]

If we consider transparent mathematical truths in themselves, not e.g. in so far as they are produced by an individual's calculations, their necessary nature defeats any sceptical challenge based on the conceivability of a mistake or on the mere logical possibility of their negation, which is not only unthinkable, but also a contradiction *in terminis*. There remains only one problem: God's omnipotence and His possible maliciousness.

The Malicious Demon Argument (MDA) represents the last sceptical benchmark through which we may attempt to cast a shadow over the total transparency of simple mathematical truths and hence their total certainty. Initially, this may seem to be an impossible task. (DA) exploits the logical possibility that one may be constantly dreaming in order to challenge the whole class of reliable empirical beliefs. The next stage in the escalation is represented by the dogmatic exploitation of the logical necessity of mathematical truths. We have seen that some mathematical truths are opaque and can be challenged by (FA), but those which are transparent present no problem in terms of access to and acknowledgement of their necessary status. The dogmatist seems to have made the last modal step and to have left the sceptic no room for any further escalation.

This sense of a closure of the sceptical dialectic, caused by the absence of a further modal level beyond logical necessity, is nevertheless misleading. In order to challenge the certainty of transparent and necessary mathematical truths, it is not necessary to take a vertical step up, because a modal escalation does not necessarily have to be intensional, but can also be extensional. The intensional escalation goes from the actual possibility of (OM) to the logical possibility of (DA) and (FA), to the logical impossibility of \neg (TT). It is hierarchical and its closure brings the sceptical analysis to the verge of extensional escalation. The latter consists in the “horizontal” elimination of any constraint in the use of the concept of logical possibility, i.e. in the passage from a predicative to an impredicative use of the concept of possibility. In general, an impredicative concept or definition presupposes or conceptually involves a totality to which the *definiendum* itself belongs.⁸ In the specific case represented by the benchmark for transparent truths, the impredicative use of the concept of possibility leads to the adoption of a definition of omnipotence as absolutely unlimited power.

During the first three stages of the sceptical escalation, the concept of possibility has been used in a predicative way. The first three arguments show that one-to-one logically consistent counterfactuals are constructible: given P, a corresponding benchmark shows that it is always logically possible to wonder whether \neg P may be the case. However, (MDA) cannot merely be extended to challenge transparent truths as well, for no appeal to logically possible mistakes in the generation of (TT) is possible, nor is it possible to construct \neg (TT) that are logically consistent. In this case, what is still theoretically constructible—what is still conceivable as logically possible, at least according to Descartes—is a metaphysical hypothesis that can act as the premise of a logically possible inference to the effect that no logically necessary truth is ever absolutely reliable. This is the Malicious Demon Argument:

[MDA] And yet firmly rooted in my mind is the long-standing opinion that there is an omnipotent God who made me the kind of creature I am. How do I know that he has not brought about it that there is no earth, no sky, no extended thing, no shape, no size, no place, while at the same time ensuring that all these things appear to me to exist just as they do now? [...] I am finally compelled to admit that there is not one of my former beliefs about which a doubt may not properly be raised. [...] I will suppose therefore that not God, who is supremely good and the source of truth, but rather some malicious demon of the utmost power and cunning has employed all his energies in order to deceive me. I shall think that the sky, the air, the earth, colours, shapes, sounds and all external things are merely the delusions of dreams which he has devised to ensnare my judgements. [AT VII, 21-22; CSM II, 14-5]

(MDA) could be formulated in terms of Fate, a bizarre God, unkind Nature, random Natural Selection, a Crazy Scientist and the Brain-in-a-vat hypothesis, or a Malicious Demon. It does not matter which special ontology it relies on, as long as the concept of possibility is used reflectively, in the following way:

1. there is no logical contradiction in assuming that the source x of the hyperbolic doubt may

⁸ More precisely, an impredicative definition of a concept or an entity is one in terms that require quantification ranging over a set of properties that includes that which is to be defined, e.g. Socrates is A, where A = “someone having all the properties of a great philosopher”, where one of the properties so ascribed must be A itself.

be absolutely omnipotent (AO(x)) and malicious (MD(x))

2. absolute omnipotence is defined impredicatively: if AO(x) is the case, then x can invert the truth value of any well-formed formula ϕ (we can symbolise this by means of the inversion alethic function $f^{-A}(x, \phi)$; a well-formed formula is any declarative content expressed in the language of the encyclopedia) and hence, more specifically, it can bring it about that *any* truth—including transparent mathematical truths such as $2 + 3 = 5$ —may be false.

Note that the interpretation of absolute omnipotence given in (2) is based on the view that, according to Descartes, nothing is absolutely necessary or impossible for God, so even eternal truths, i.e. necessary truths like mathematical theorems, are necessary not in the sense that not even God could change them, but in the sense that, although God could change them, because he is absolutely omnipotent, it is certain that God will not wish to change them, because he is absolutely perfect, benevolent and truthful (see the Appendix for a discussion of the connection between the problem of the creation of eternal truths and (MDA)).

Here is a more formal way of formulating (MDA), where x ranges over the domain of possible entities, ϕ ranges over the domain represented by the contents of the encyclopedia, and f^{-A} is constructed following the definition of an ordinary inverse function,⁹ and should not be confused with the negation function:

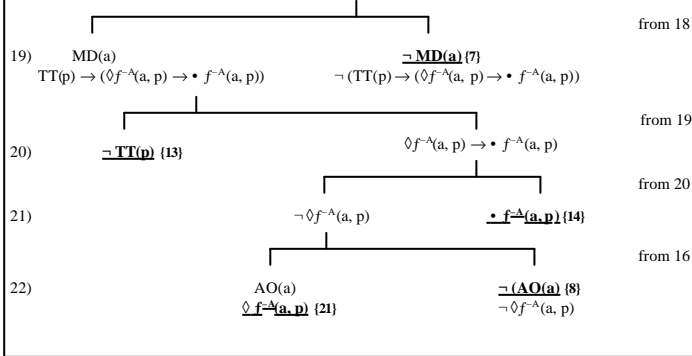
⁹ Given a function $f: A \rightarrow B$, a function $f^{-1}: B \rightarrow A$ is called a *left inverse* for f if and only if, for all a in A, $f^{-1}(f(a)) = a$, and a *right inverse* if and only if, for all b in B, $f(f^{-1}(b)) = b$. The function is simply *inverse* if both conditions hold. Only an injection has a left inverse, only a surjection has a right inverse and only a bijection has inverses f^{-1} , where $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. A function is bijective or a bijection or a one-to-one correspondence if it is both injective (no two values map to the same value) and surjective (for every element of the codomain there is some element of the domain which maps to it), i.e. there is exactly one element of the domain which maps to each element of the codomain.

The Malicious Demon Argument: a Formal Analysis

$\forall x \forall \phi (AO(x) \leftrightarrow \hat{\phi} f^{-\Lambda}(x, \phi)), \forall x \forall \phi (MD(x) \leftrightarrow (TT(\phi) \rightarrow (\hat{\phi} f^{-\Lambda}(x, \phi) \rightarrow \bullet f^{-\Lambda}(x, \phi))))$
 $\models (\exists x (MD(x) \wedge AO(x))) \rightarrow \forall \phi \exists y (TT(\phi) \rightarrow \bullet f^{-\Lambda}(y, \phi))$

c.e.s.

- 1) $\forall x \forall \phi (AO(x) \leftrightarrow \hat{\phi} f^{-\Lambda}(x, \phi))$
- 2) $\forall x \forall \phi (MD(x) \leftrightarrow (TT(\phi) \rightarrow (\hat{\phi} f^{-\Lambda}(x, \phi) \rightarrow \bullet f^{-\Lambda}(x, \phi))))$
- 3) $\neg (\exists x (MD(x) \wedge AO(x))) \rightarrow \forall \phi \exists y (TT(\phi) \rightarrow \bullet f^{-\Lambda}(y, \phi))$
- 4) $\exists x (MD(x) \wedge AO(x))$ from 3
- 5) $\neg \forall \phi \exists y (TT(\phi) \rightarrow \bullet f^{-\Lambda}(y, \phi))$ from 3
- 6) $MD(a) \wedge AO(a)$ x = a from 4
- 7) $MD(a)$ from 6
- 8) $AO(a)$ from 6
- 9) $\exists \phi \neg \exists y (TT(\phi) \rightarrow \bullet f^{-\Lambda}(y, \phi))$ from 5
- 10) $\neg \exists y (TT(\phi) \rightarrow \bullet f^{-\Lambda}(y, \phi))$ from 9
- 11) $\forall y \neg (TT(\phi) \rightarrow \bullet f^{-\Lambda}(y, \phi))$ from 10
- 12) $\neg (TT(\phi) \rightarrow \bullet f^{-\Lambda}(a, \phi))$ from 11
- 13) $TT(\phi)$ from 12
- 14) $\neg \bullet f^{-\Lambda}(a, \phi)$ from 12
- 15) $\forall \phi (AO(a) \leftrightarrow \hat{\phi} f^{-\Lambda}(a, \phi))$ from 1
- 16) $(AO(a) \leftrightarrow \hat{\phi} f^{-\Lambda}(a, \phi))$ from 15
- 17) $\forall \phi (MD(a) \leftrightarrow (TT(\phi) \rightarrow (\hat{\phi} f^{-\Lambda}(a, \phi) \rightarrow \bullet f^{-\Lambda}(a, \phi))))$ from 2
- 18) $MD(a) \leftrightarrow (TT(\phi) \rightarrow (\hat{\phi} f^{-\Lambda}(a, \phi) \rightarrow \bullet f^{-\Lambda}(a, \phi)))$ from 17



Premise (1), $\forall x \forall \phi (AO(x) \leftrightarrow \diamond f^{-A}(x, \phi))$, defines the property of absolute omnipotence. It states that x is absolutely omnipotent if and only if it is possible for x to invert the truth value of any formula. Of course, from (1) it follows that ϕ may also be a TT. Premise (2), $\forall x \forall \phi (MD(x) \leftrightarrow (TT(\phi) \rightarrow (\diamond f^{-A}(x, \phi) \rightarrow \ddot{y} f^{-A}(x, \phi))))$, defines the property of absolute maliciousness: it states that x is a malicious entity if and only if, if it is possible for x to invert the truth value of any transparent truth, then x necessarily inverts the truth value of any transparent truth. So when transparent truths are in question, the malicious demon *seems* to have no freedom, but more on this possible inconsistency later. The conclusion states that from (1) and (2) it follows that, if there is an omnipotent and malicious entity, then any transparent truth is a falsehood. (MDA) calls now for at least four comments.

1) The impredicative nature of (MDA) is clear from premise (1): x is AO if and only if, for any well-formed formula ϕ in our language, x can invert its truth value, including the same formula that states this, i.e. $\diamond \forall \phi (TT(\phi) \rightarrow f^{-A}(x, \phi))$. The definition of “absolute omnipotence” turns out to range over an illegitimate totality. I shall return to this point when briefly illustrating various possible solutions of (MDA).

2) The extensional escalation caused by the impredicative definition of the concept of absolute omnipotence makes it clear that what is now logically possible is the formula $\exists x \forall \phi \ddot{y} f^{-A}(x, \phi)$, where x is absolutely omnipotent and malicious, not simply $\neg \phi$, whenever ϕ is not a transparent mathematical truth, that is $\forall \phi (\neg TT \phi \rightarrow \diamond \neg \phi)$, as is the case in the course of the intentional escalation. The importance of this indirect strategy is repeatedly stressed by Descartes, whenever he speaks about the inability to doubt beliefs that appear as certain as one may wish them to be (cf. AT VII, 22-23; CSM II, 14-5 for a first statement).

3) Descartes interprets (MDA) as an argument that not merely forces us to doubt the truth value of TT, but asserts, *conditionally*, that, if there is an omnipotent and malicious source of doubt, and if that source is actively engaged in misleading us, then all propositions that we perceive as transparent truths of mathematics are actually false, while we may further suppose that all transparent falsehoods of mathematics are actually true.

In view of this [i.e. habitual opinions keeping coming back], I think it will be a good plan to turn my will in completely the opposite direction and deceive myself, by pretending for a time that these former opinions [here Descartes is referring not only to all his opinions in general, but especially to his transparent truths] are utterly false and imaginary. [AT VII, 22; CSM II, 15] [...] I will suppose then, that everything I see is spurious. I will believe that my memory tells me lies, and that none of the things that it reports ever happened. I have no sense. Body, shape, extension, movement and place are chimeras. [AT VII, 24; CSM II, 16]

Descartes’ reading of f^{-A} as a function that, based on the impredicative definition of absolute omnipotence, inverts the truth value of ϕ but leaves untouched its modal nature, is *contextually* correct in two different senses: hermeneutically, it is consistent with Descartes’ natural acceptance of a binary type of logic; theoretically, it is consistent with the kind of possibly misleading logic that the malicious demon may have given us. For the proper interpretation of the truth table of $f^{-A} \phi$ is:

- a) $f^{-A} \diamond \phi \leftrightarrow \diamond \neg \phi$
- b) $f^{-A} \diamond \neg \phi \leftrightarrow \diamond \neg \neg \phi = \diamond \phi$
- c) $f^{-A} \ddot{y} \phi \leftrightarrow \ddot{y} \neg \phi$
- d) $f^{-A} \ddot{y} \neg \phi \leftrightarrow \ddot{y} \neg \neg \phi = \ddot{y} \phi$

A certain lack of freedom is what we would expect in the description of the nature of the malicious demon (the demon cannot help being but malicious), which is portrayed as a sort of carbon copy of God, and in the quotation above, clause (c) is precisely what Descartes “correctly”—i.e. correctly according to the logic that the malicious demon may have given us to work with—offers as a proper interpretation of the assumption that there may be an

absolutely malicious and omnipotent God.¹⁰ If an entity is both absolutely omnipotent and malicious then, according to our logic, if that entity means to mislead us, we can rationalise this by saying that it can only do so by necessarily inverting the alethic value of any transparent truth. (MDA) cannot state that transparent truths are just unconditionally doubtful—as happens in the previous three arguments during the intensional escalation—because this would require the following truth table for f^{-A} :

- i) $f^{-A} \diamond \phi \leftrightarrow \diamond \neg \phi$
- ii) $f^{-A} \diamond \neg \phi \leftrightarrow \diamond \neg \neg \phi = \diamond \phi$
- iii) $f^{-A} \dot{\forall} \phi \leftrightarrow \dot{\forall} \phi$
- iv) $f^{-A} \dot{\forall} \neg \phi \leftrightarrow \dot{\forall} \neg \phi$

and given the *standard* modal equivalences ($\diamond \phi \leftrightarrow \neg \dot{\forall} \neg \phi$) and ($\dot{\forall} \phi \leftrightarrow \neg \diamond \neg \phi$), in (iii) and (iv) the function would collapse the concept of logical necessity and that of logical possibility, generating an inconsistency.¹¹

Of course, one may object that, since the question concerns the very nature of modalities themselves, the fact that the previous reasoning would be contradictory may not represent an insurmountable problem. If there is a malicious God who is indeed misleading us, he may be misleading us into believing that he has no freedom but to invert the truth-value of necessary truths. This is why we need to add the proviso “according to our logic”.

We may further remark that Descartes’ reading of f^A engenders no epistemological problem, contrary to what Bourdin argues, rather naively:

Since it does not seem certain that two and three make five, and since the above rule [i.e. the reading of f^{-A} as a function that inverts the truth value of ϕ but leaves untouched its modal nature] obliges me to say and believe that two and three do not make five, may I ask whether I should constantly believe this, to the extent of convincing myself that it is certain and cannot be otherwise? [...] Do you mean to regard it as certain that two and three do not make five? Do you mean this to be certain, and to appear as certain to everyone—so certain that it is safe even from the tricks of the evil demon? [...] Will our statement [i.e. two and three do not make five] be doubtful and uncertain, just like the statement that two and three make five? If so, if the statement that two and three do not make five is doubtful, then following your rule I will believe and state that it is false and I will assert the opposite: I will assert that two and three do make five. [AT VII, 458; CSM II, 307]

Bourdin is obviously mistaken. The certainty concerning the truth of TT is not transformed into the certainty of their falsehood, since we do not and cannot know whether there is an omnipotent source of doubt and when or whether it is actively misleading us. The malicious demon, if he exists, plays hide and seek with the “I” of the *Meditations*, or as Descartes himself puts it:

When I said that doubtful matters should for a time be treated as false, or rejected as false, I merely meant that when investigating the truths that have metaphysical certainty we should regard doubtful matters as not having any more basis than those which are quite false. [AT VII, 461; CSM II, 309] [...] What I said was that doubtful items should not be regarded as having any more basis than those which are wholly false; but this was so as to enable us to dismiss them completely from our

¹⁰ Bréhier, “The Creation of the Eternal Truths in Descartes’ System”, p. 200 writes: “Hence, to play his part, the evil genius—that imaginary being on whom Descartes has conferred the omnipotence of God without His goodness—must be the creator of essences, a creator who is completely arbitrary and continually changing”. Only the first half of the statement is fully justified. If the arbitrary and changing nature of the malicious demon implies that transparent mathematical truths may, by chance, happen to be true, then the second half of the statement is mistaken.

¹¹ The contradiction can be summarised thus:

- 1) From (i) and (iv): $f^{-A} \diamond \phi \leftrightarrow \diamond \neg \phi \leftrightarrow f^{-A} \dot{\forall} \neg \phi$
- 2) from ($\diamond \phi \leftrightarrow \neg \dot{\forall} \neg \phi$): ($\neg \diamond \phi \leftrightarrow \dot{\forall} \neg \phi$);
- 3) from (2): $f^{-A} \neg \diamond \phi \leftrightarrow f^{-A} \dot{\forall} \neg \phi$ (if $p \leftrightarrow q$ then inverting the truth-value of p means inverting the truth value of q), therefore
- 4) from (1) and (3): $f^{-A} \diamond \phi \leftrightarrow f^{-A} \neg \diamond \phi$, which is an obvious contradiction in a two-value logic.

thought, and not so as to allow us to affirm first one thing and then its opposite. [AT VII, 462; CSM II, 310]

(MDA) generates a conditional, (the position of the previous comma is crucial) total-uncertainty because a universe containing $(AO(x) \wedge MD(X))$ appears to be closed under f^{-A} , i.e. it appears to contain every member of the set produced by f^{-A} acting on members of the set. Since a Malicious Demon has the power to transform any truth into a falsehood and any falsehood into a truth, Descartes is right in saying that, after the introduction of (MDA), he can have no certainty whatsoever, either empirical or mathematical.

4) A final remark. By mathematical sciences Descartes means sciences whose truths concern

corporeal nature in general, and its extension; the shape of extended things, the quantity, or size and number of these things; the place in which they may exist, the time through which they may endure [the place where they are, the time which measures their duration, (French version)], and so on. [AT VII, 21; CSM II, 14]

It is important to stress this “realistic” nature of Descartes’ conception of mathematics. When Descartes is talking about the Malicious Demon’s capacity to deceive him in matters concerning

the sky, the air, the earth, colours, shapes, sounds and all external things [AT VII, 22-23; CSM II, 15]

and then even his body, he is actually restating, in more vivid terms, a point previously made with respect to God and his capacity to bring it about that

there in no earth, no sky, no extended thing, no shape, no size, no place, while assuring at the same time that all these things appear to me to exist just as they do now. [AT VII, 21; CSM II, 14]

The hypothesis that an omnipotent God may change even what seems to be necessarily the case refers generally to any form of knowledge but includes, more significantly, the mathematical knowledge we may have of such phenomena, the “transparent truths” that are still indubitable after the employment of (DA).¹² Thus, there can be no doubts that (MDA) does indeed address primarily the problem of mathematical scepticism about transparent truths. We have seen that Descartes fully clarifies this point in the Third Meditation (AT VII, 36; CSM II, 25, see quotation above). If the problem posed by (MDA) is not solved, no matter how unlikely the hypothesis that there may be a malicious God, still mathematical knowledge will never be sufficiently reliable to satisfy the twofold principle of maximum safety and hence provide the encyclopedia with an absolutely certain foundation. The difference between empirical and mathematical sciences is based on the higher level of certainty achievable by the latter owing to their subjects and transparency, but we need to keep in mind that, for Descartes, this difference is not an epistemological dichotomy.¹³ At the end of the *Meditations*, both empirical and mathematical knowledge are certain, true and informative descriptions of the intrinsic nature of real or possible states of the world. For Descartes, mathematics and in particular geometry, which he considers the model for all knowledge (*Discourse* Part II, AT VI, 19; CSM I, 120), is not a system of tautologies, logically necessary because they are analytically necessary, or

¹² Here I depart from a common interpretation of the passage, according to which “As Gouhier has also noted, the summary of ‘doubts’ in the concluding passage just quoted does not include mention of mathematical propositions—which are not again brought into discussion until the Third Meditation”, Margaret D. Wilson, *Descartes* (London: Routledge, 1982), p. 33. The latter interpretation presents two main problems: (a) as its supporters are ready to acknowledge, it implies a macroscopic lack of uniformity in the treatment of mathematical scepticism within the *Meditations*, (b) it is based on a treatment of mathematical truths and logical principles as belonging to the same class of ontologically non-committed propositions. If (a) is very implausible, (b) appears to be untenable because of Descartes’ conception of mathematics as a science of the empirically possible.

¹³ For a description of mathematics as a general science of order and measure see AT X, 377-8; CSM I, 19.

necessary in virtue of their logical forms, but a body of metaphysically necessary truths (cf. AT VII, 461; CSM II, 309), i.e. truths that are absolutely necessary because they describe the essential and immutable (i.e. established and mutable only by God) nature of whatever is or can be real, including the physical world. In other words, arithmetic and geometry are the grammar of the discrete and continuum domains. God could change the grammar, if he so wished, but in that case he would also provide us with a mind capable of understanding the new language, because he is not a deceiver. I shall return to this point in the next two sections, when discussing briefly the Humean approach to (MDA) and the puzzling absence of any Pyrrhonian argument in Descartes' discussion of mathematical scepticism.

Three Non-Cartesian Approaches to the Malicious Demon Argument

Before analysing the Cartesian solution of (MDA), it is worth surveying three alternative approaches. For the sake of simplicity, I shall refer to them as the Humean, the Quinean and the Russellian approach.

The Humean approach to (MDA) may be traced to Locke and develops through the logical positivists, Wittgenstein's *Tractatus* and A. J. Ayer's *Logic Truth and Language*. It consists in savaging the certainty of mathematics by referring to the use of conventional rules, at the expense of its epistemic and ontologically informative value, and hence of the immediate explicability of mathematics' scientific effectiveness and technological applicability. Mathematical statements, such as $2 + 3 = 5$ or "a square has no more than four sides", are necessarily true and indubitable, but only because they reflect the way in which we use a particular language's vocabulary and grammar. In other words, mathematical knowledge consists in conventional tautologies that say nothing about the world and are necessarily true because analytic. It is not the task of this paper to develop a criticism of this conventionalist approach to mathematical knowledge. Suffice it to remark that this is definitely alien to Descartes' philosophy, according to which nature is an embodiment of mathematics, and may save mathematical knowledge from the (MDA)—although there remains the problem of what makes semiotic conventions not merely applicable (for this is still explicable in terms of some instrumentalist selection), but fully reliable, assuming that a conventionalist interpretation of mathematics is correct—only insofar as it makes mathematical statements no longer a matter of discussion within the Cartesian foundationalist project. Precisely because it can avoid the application of (MDA) to mathematical certainties, the conventionalist approach is perfectly consistent with a sceptical philosophy à la Hume.

A slightly stronger approach consists in acknowledging the problem raised by (MDA) and the fallibility of mathematical knowledge, but then discarding the twofold principle of maximum safety as too demanding. In this way it becomes possible to accept a moderately more fallibilist philosophy of mathematics. This Quinean solution is obviously in contrast to Descartes' foundationalist project, from which perspective it amounts to accepting defeat in the search for ultimate certainty.

A third approach consists in blocking the sceptical escalation, making any extensional development an "illegal" move. This means banning any viciously circular impredicative definition or concept, including the impredicative definition of absolute omnipotence, by imposing a kind of hierarchical formal system whereby what is logically necessary can in principle exercise its power only on lower types of modalities but not on what is equally logically necessary; what is real, but contingent, can in principle exercise its power on anything but the higher class represented by what is logically necessary; and what is only possible can exercise no effective power. This alternative, which has its roots in the medieval debate about God's omnipotence (thus Thomas Aquinas argued that necessary truths are not subject to change according to God's will, cf. *Contra Gentiles*, Book II, ch. 25) and is obviously reminiscent of Russell's theory of types, has the advantage of eliminating the vicious-circle

antinomy involving an omnipotent entity which is so powerful as to be able to transform into a falsehood even the truth that it is omnipotent. Its disadvantage, however, is that it is an *ad hoc* solution at best, and it requires a whole ontological scenario at worst. In the former case, the Cartesian view may simply be that it is not quite clear why we should stop the road of inquiry by a *fiat*. It is not immediately obvious that unintelligibility may work as a fundamentally satisfactory criterion for limiting God's omnipotence, and Descartes repeatedly argues in favour of the opposite view. In the latter case, any theory about omnipotence and modal hierarchies seems to be as ontologically demanding as the simple elimination of the Malicious Demon hypothesis. It is not by chance that, *mutatis mutandis*, similar criticisms apply to Russell's logicist foundationalism.

The Mathematical Atheist: Don Juan on the Solution of the Malicious Demon Argument.

The last alternative is provided by a further escalation of the self-referential nature of the dialectical process. If the sceptic is allowed to use the concept of possibility impredicatively in (MDA), the dogmatist can also further increase the escalation of his search for ultimate certainty extensionally, and look for a necessary truth that may be brought about, self-referentially, by the very process that tries to challenge it.

As we know, the *cogito* is such a certainty. Its nature is such that any further testing can only reinforce it. This peculiar feature of the *cogito* is made possible by its empirical and mathematical emptiness. Were the solution of (MDA) to consist in, or convey any empirical or mathematical truth, it would be challengeable precisely on that ground.

The emptiness of the *cogito*, however, is not only its strength but also its weakness. In the same way that the sceptic cannot test transparent mathematical truths directly, by appealing to the logical possibility of their falsehood, but needs to construct a metaphysical hypothesis, so that he can indirectly infer that even transparent mathematical truths could be converted into necessary falsehoods by an absolutely omnipotent source of doubt, and are so converted by an omnipotent malicious demon, likewise the dogmatist cannot simply rely on the *cogito* to reconstruct the whole encyclopedia, but requires an indirect solution, a metaphysical bridge that may join certainty and truth. The *cogito* is absolutely certain, its very certainty has its source in the logical possibility of doubting, but it cannot represent a *direct* guarantee of knowledge, for it is content-empty and can only provide a formal criterion for selecting equally certain extensions of the encyclopedia, all beliefs that turn out to be as clear and distinct as the *cogito* itself. To transform an encyclopedia of certainties into an encyclopedia of truths, to move from internal, absolute but only formal indubitability to external, empirical and total reliability, Descartes requires a further ontological bridge, which grants that anything that is absolutely certain is true, and any possible truth will be, if known, as certain as the *cogito*. Certainty and truth *conventuntur* but it is possible to establish that they do so only indirectly, if we can prove, on the basis of the *cogito*, that there is a benevolent, veridical and omnipotent warrant, God, who guarantees their intrinsic connection. If it is possible to prove the existence of an omnipotent, benevolent and truthful God then:

This disposes of the most serious [i.e. radical] doubt [the MDA] which arose from our ignorance about whether our nature might not be such as to make us go wrong even in matters which seemed to us utterly evident. Indeed, this argument [i.e. the existence of God] easily demolishes all the other reasons for doubt which were mentioned earlier. Mathematical truths should no longer be suspect, since they are utterly clear to us. [AT IXB, 16-7, see also 328-9; CSM, I, 203, see also 290-1]

One may then read the *cogito* as a proof¹⁴ of the inconsistency of the existence of a malicious

¹⁴ The reasoning may be formalised thus (note that a more careful analysis could be provided in terms of

and omnipotent source of doubt, in favour of the presence of a benevolent, omnipotent source of universal uniformity and intelligibility of whatever can be real. The *cogito* is the subjective side of the new foundation, but without an external guarantee, there can be only human certainties but no absolute truths, and hence no utterly indubitable knowledge of the intrinsic nature of reality. It follows that the principal difficulty in the *Meditations* turns out to be the proof for the existence of God and the consequent Cartesian circle: Descartes is in need of a first indubitable extension of *knowledge* in order to be able to prove the actual existence of God, but requires the existence of God as a metaphysical principle in order to prove that a first indubitable extension of knowledge is possible.

The Cartesian failure to solve satisfactorily the problem of an external anchor for the system of subjective certainties is one of the problems at the roots of the history of modern epistemology. I have analysed the problem of the Cartesian circle elsewhere,¹⁵ and it is not worth entering into the discussion of its nature in this context. What is important to stress here, in connection with the analysis of mathematical scepticism, is that, given the nature of (MDA) and its solution by means of the *cogito*, Descartes concludes, at the end of the Fifth Meditation, that there can be no mathematical knowledge without a metaphysical guarantee:

Now, however, I have perceived that God exists, and at the same time I have understood that everything depends on him, and that he is no deceiver; and I have drawn the conclusion that everything which I clearly and distinctly perceive is of necessity true. Accordingly, even if I am no longer attending to the arguments which led me to judge that this is true, as long as I remember that I clearly and distinctly perceived it, there are no counter-arguments which can be adduced to make me doubt it, but on the contrary I have true and certain knowledge of it. And I have knowledge not just of this matter, but of all matters which I remember ever having demonstrated, in geometry and so on. [...] Thus I see plainly that the certainty and truth of all knowledge depends uniquely on my awareness of the true God, to such an extent that I was incapable of perfect knowledge about anything else until I became aware of him. And now it is possible for me to achieve full and certain knowledge of countless matters, both concerning God himself and other things whose nature is intellectual, and also concerning the whole of that corporeal nature which is the subject-matter of pure mathematics [and also concerning things which belong to corporeal nature in so far as it can serve as the object of geometrical demonstrations which have no concern with whether that object exists, (French version)] [AT VII, 70-1; CSM II, 48-9]

This is the point against which both Mersenne and Gassendi move their *Objections*,¹⁶ in three different versions:

[Version A] Moreover, an atheist is clearly and distinctly aware that three angles of a triangle are equal to two right angles; but so far is he from supposing the existence of God that he completely denies it. [*Second Set of Objections*, AT VII, 125; CSM II, 89]

[Version B] Finally, you say that the certainty and truth of all knowledge depends uniquely on your knowledge of the true God, so that without such knowledge no true certainty and knowledge is attainable. [...] I do not think you will find it easy to make anyone believe that before you established the above conclusion about God [i.e. the proof of his existence] you were less certain of these geometrical proofs than you were afterwards. These proofs certainly seem to be so evident and certain that they compel our assent by themselves, and once they have been perceived they do

inversion of the level of certainty, but this refinement does not change the general conclusion):

$TT(\phi = \text{cogito})$

$\forall x \forall \phi (TT(\phi = \text{cogito}) \rightarrow \neg \diamond f^{-A}(x, \phi))$

$\exists x \forall \phi (MD(x) \rightarrow (TT(\phi) \rightarrow \ddot{y} f^{-A}(x, \phi)))$

$\models \neg \exists x MD(x)$

¹⁵ *Scepticism and the Foundation of Epistemology—A Study in the Metalogical Fallacies* (Brill: Leiden, 1996).

¹⁶ *Second Set of Objections*, compiled by Mersenne, AT 125; CSM II, 89, *Reply*, AT 141; CSM II, 101; Gassendi's *Fifth Set of Objections*, AT 326-328; CSM II, 227-8, *Reply*, AT 384; CSM II, 263-4; *Sixth Set of Objections*, compiled by Mersenne, AT 414-5; CSM 279, *Reply*, AT 428; CSM II, 289.

not allow our intellect to remain further in doubt. So, indeed, when faced with these proofs, the mind may very well tell the evil demon to go hang himself, just as you yourself emphatically asserted that you could not possibly be deceived about the proposition or inference “I am thinking, hence I exist”, even though you had not yet arrived at knowledge of God. Of course it is quite true—as true as anything can be—that God exists, is the author of all things, and is not a deceiver; but these truths, seem less evident than the geometrical proofs, as is shown by the fact that many people dispute the existence of God, the creation of the world, and so on, whereas no one impugns the demonstrations of geometry. In view of this, is there anyone whom you will convince that the geometrical proofs depend for their evidence and certainty on the proofs concerning God? Surely no one imagines that such atheists as Diagoras or Theodorus cannot be made completely certain of these geometrical proofs. And how often do you find a believer who, if he is asked why he is certain that the square of the hypotenuse of a right-angled triangle is equal to the squares on the other sides, will answer: “Because I know God exists and cannot deceive, and that he is the source of this geometrical truth and of all other things?” Will he not answer instead: “Because I know it and am convinced of it by an indubitable demonstration?” And how much more likely is it that Pythagoras, Plato, Archimedes and Euclid and the other mathematicians will answer in this way? For none of them seems to have thought about God in order to make himself completely certain of his demonstrations! [*Fifth Set of Objections*, AT VII, 327-8; CSM II, 227-8]

[Version C] The *fourth* difficulty concerns the kind of knowledge possessed by an atheist. When the atheist asserts “If equals are taken from equals the remainders will be equal” or “The three angles of a rectilinear triangle are equal to two right angles” and numerous similar propositions, he maintains his knowledge is very certain and indeed—on your criterion—utterly evident. For he cannot think of these propositions without believing them to be wholly certain. He maintains that this is so true that even if God does not exist and is not even possible (as he believes), he is just as certain of these truths as if God really existed. Moreover he maintains that no reason for doubt can be presented to him which could shake him in the slightest or make him at all uncertain. [*Sixth Set of Objections*, AT VII, 415; CSM II, 279]

It would be surprising if, in his brief replies, Descartes were to provide utterly new arguments to support the impossibility of overcoming mathematical scepticism in the absence of God. His position can be interpreted, more correctly and satisfactorily, as a consistent defence and further explanation of what he has already argued in the *Meditations*.

On this methodological ground, it becomes possible to show that Descartes offers the following arguments against the epistemological justification of Don Juan’s mathematical atheism.

1. Reply to version A in the Second Set of Replies: a restatement of (FA) and (MDA), where both arguments are further supported by an inductive argument about the extension of time. In the first half of his objection, Mersenne (if he is the author, and not just the compiler, of this specific argument) appears to have missed the nature of sceptical escalation and the self-referential peculiarity of the *cogito*. As far as the problem of mathematical scepticism is concerned, Descartes acknowledges that Don Juan’s doxastic position is indisputable:

an atheist can be ‘clearly aware that the three angles of a triangle are equal to two right angles’ [AT VII, 141; CSM II, 100]

but his awareness cannot count as knowledge since it can be rendered doubtful. Descartes’ reply is rather brief, and does not explicitly mention how the geometrical theorem in question can be rendered doubtful. We are only told that, since Don Juan is an atheist

he cannot be certain that he is not being deceived on matters which seem to him to be very evident (as I fully explained). And although this doubt may not occur to him, it can still crop up if someone else raises the point or if he looks into the matter himself. So he will never be free of this doubt until he acknowledges that God exists. [AT VII, 141; CSM II, 100]

Either the geometrical theorem is treated as an opaque truth, in which case (FA) applies, or it is treated, more likely, as a transparent truth, but then (MDA) applies: in either case, Don Juan is

unjustified in saying that he knows that the theorem is true. Note that, in the reply, Descartes does not mention any intentional issue related to the fallibility of memory, but reinforces his arguments with an extensional and inductive challenge: the fact that, in the past, the atheist may have never come across sufficient doubts to question his mathematical certainties does not mean that these are absolutely certain and may not be undermined by new doubts in the future.

The interpretation just provided differs from the suggestion that Descartes may be distinguishing

here between an isolated cognition or act of awareness (*cognitio*) and systematic, properly grounded knowledge (*scientia*).¹⁷

which, though correct and relevant, does not capture the essential point. It is true that a single certainty, *hic et nunc*, cannot be connected with any other certainty without running into the sceptical problem stated by (FA), but in his reply Descartes is not, actually could not be, merely arguing that the atheist cannot justify his faith in the whole of arithmetic or geometry as sciences, for this would be an actual defeat of his view that only the *cogito* represents an indubitable ground for the reconstruction of the encyclopedia. Consistently with his own position, Descartes is rather trying to argue that, even if the atheist seems to be clearly and distinctly perceiving a mathematical theorem, he cannot treat that theorem as knowledge because of (MDA).

2. Reply to version B in the Fifth Set of Replies: implicit restatement of (FA) and (MDA), where both arguments are further supported by an inductive argument about the extension of the population of knowers.

In this case, Descartes simply reminds the reader that

The next points you raise are ones which I have already adequately answered elsewhere. [AT VII, 384; CSM II, 263]

He adds, however, two interesting remarks. First, he further reinforces his arguments inductively by stressing that, in the search for the absolutely certain foundation of knowledge, it is only what is logically prior that matters, not what is ordinarily more intuitive, as Gassendi is forced to assume on the basis of his fallibilist position:

one thing [i.e. a mathematical truth] is not proved to be better known than another [i.e. God's existence] just because a greater number of people think it is true. What shows it to be better known is simply that those who know the true nature of both things see that it is prior in the order of knowledge and more evident and certain. [AT VII, 384; CSM II, 263]

The second point made by Descartes is that, against the authors cited by Gassendi

I cite the sceptics who did have doubts about these very geometrical demonstrations. And I insist that they could not have done so had they known the true nature of God. [AT VII, 384; CSM II, 263]

Descartes does not tell us who these sceptics are, but it is likely that he had in mind Sextus Empiricus' writings, especially *Adversus Mathematicos* III and IV, i.e. *Contra Geometras* and *Contra Arithmeticos*, which had been translated into Latin by Gentianus Hervetus in 1569. Descartes objects, against Gassendi, that even ancient philosophers had entertained doubts about the certainty of mathematics. This appeal to ancient authorities is in itself scholarly trivial and certainly of no philosophical consequence, but it raises an intriguing question: seeing that Descartes was obviously aware that ancient sceptics had mounted a vigorous attack against

¹⁷ *Meditations on First Philosophy*, tr. by John Cottingham, CSM II, 101, note 2, the note continues "Compare the remarks in *The Search for Truth* about the need to acquire 'a body of knowledge firm and certain enough to deserve the name 'science'', below p. 408; see also p. 104 below, and *Rules*, vol. I, pp. 10ff.". The same point is restated in J. Cottingham, *A Descartes Dictionary* (Oxford: Blackwell, 1993), p. 18-19.

mathematical knowledge, why did he never employ any of their arguments in his method of doubt? One can only speculate that he might not have found them sufficiently convincing to be worth using, and in this case, one may further suspect that the theoretical reason for Descartes' complete disregard for the Pyrrhonian approach to mathematical scepticism lies in his philosophy of mathematics.¹⁸

Above, we have started to see Descartes' realist conception of mathematics. Mathematics provides quantitative, rigorous, exact and effective knowledge about reality, while possessing at the same time a self-evident basis. As a body of knowledge, it enjoys the highest degree of logical certainty and its basic notions are epistemologically more intuitive than other notions, yet arithmetic and geometry are not logico-axiomatic systems to be interpreted according to specifiable domains, but bodies of a priori true, informative propositions about what there is and what is possible. Descartes' realism is Platonist (A priori Innatism), not Euclidean or Aristotelian (Abstractionism): mathematics consists of universal knowledge of eternal, immutable and necessary properties of whatever is or may be real, and it can provide an effective description of nature because nature is built according to principles which are inherently mathematical and fully intelligible, but access to mathematical knowledge is not empirical and does not proceed by means of abstraction (i.e. elimination) and idealisation (i.e. improvement) of certain properties starting from empirical perceptions. Knowledge of mathematical truths concerning formal properties (shapes, quantities, relations, e.g. ratios, etc.) is the result of internal intuition by the light of reason.

From this perspective, the Pyrrhonian position is definitely not interesting. As Leibniz remarked, in connection with mathematical scepticism, "all the difficulties raised by the Pyrrhonians concern only the empirical truths (*veritez sensibles*)", hence a Pyrrhonian version of mathematical scepticism affects primarily a Euclidean or Aristotelian view of mathematical knowledge.¹⁹ For a Pyrrhonist, non-empirical mathematics, especially geometry, is a contradiction in terms. Insofar as Euclidean geometry provides information about physical space and the behaviour of objects in it, for example, it must be true of the world and rely on a physical understanding of its primitive notions. One can then demonstrate that geometry provides no direct knowledge about the real nature of the world by determining what contradictions must necessarily arise from a physical interpretation of its elements. The process of de-physicalisation thus amounts to a process of de-alethisation of mathematics—mathematical statements in themselves are not necessarily true of the world, and a full de-alethisation of geometry amounts to showing that no geometry is possible at all. This is why, unlike Descartes' analysis of mathematical scepticism, Pyrrhonism is not mainly concerned with the certainty of mathematical knowledge but rather with the controversial ontological status of mathematical entities. Sextus Empiricus' mathematical scepticism should be interpreted as a radical use of empiricism only. For the Pyrrhonist, either mathematics counts as empirical knowledge of the world, or it is nothing at all, but as not the former, therefore it is the latter. Pyrrhonism and Platonism cannot even start disagreeing because they lack the minimal condition of possibility for dialogue, namely a reasonably similar conception of what mathematical knowledge is.

3. Reply to version C in the Sixth Set of Replies: implicit restatement of (FA) and (MDA), where both arguments are further supported by an argument about logical principles and a corollary consisting of an internal *reductio ad absurdum* of the atheist position, in terms of increasing fallibility of human knowledge.

Descartes restates once more that:

¹⁸ For more details, cf. L. Floridi, "Mathematical scepticism: a sketch with Historian in foreground", *art. cit.*

¹⁹ Letter to Edmonde Mariotte, Hälfte 1676 in G. W. Leibniz *Philosophischer Briefwechsel*, Ersten Band 1663-1685, (Darmstadt: Reichl Verlag, 1926), p. 268-269.

As for the kind of knowledge possessed by the atheist, it is easy to demonstrate that it is not immutable and certain. [AT VII, 428; CSM II, 289]

and we have seen that this can be interpreted as a reference to the combination of (FA) and (MDA). In the *Objection*, version C, we find mentioned not only a mathematical truth, but also a logical principle. Descartes does not say anything about the latter in particular, but he provides an implicit argument that further undermines the possible strength of the atheist position in the Second Set of Replies. While defending the view that the *cogito* is not a syllogism, he makes it explicit that

awareness of first principles is not normally called “knowledge” by dialecticians. [AT VII, 141; CSM II, 100]

What Descartes means is that “common notions”, i.e. self-evident principles such as “if equals are taken from equals the remainders will be equal”, may be called into doubt by an omnipotent God, if necessary (cf. *Principles of Philosophy*, AT IXB, 9-10; CSM I, 197), but, like the *cogito*, do not really qualify as part of the encyclopedia. Descartes seems to hold the view that eternal logical truths, what we would call analytic truths or tautologies, have nothing to do with actual knowledge, for they contain no ontological commitment to any degree of existence (necessary, contingent, merely possible) and hence are not “informative” about what is or can be real, but rather work like rules of inference with the logical form $\forall x (Ax \rightarrow Bx)$,²⁰ which provide the conditions of possibility of knowledge. As such, they do not represent a real issue for a sceptical analysis, which can limit itself to the critical discussion of mathematical transparent truths.

We are then provided with a further internal argument against Don Juan, an argument that becomes a corollary of (MDA) in the discussion of mathematical scepticism provided in the *Principles of Philosophy* (see above FA.4).

Suppose we consider the position of the mathematical sceptic from within. If the atheist could be convinced of the possible existence of an omnipotent Malicious Demon then, *a fortiori*, he could be convinced of God’s possible existence, but he denies the latter, and we may assume that he also rejects the former. However, even if we assume that there is no God and, *a fortiori*, no Malicious Demon, Don Juan is still not justified in believing that any transparent truth is absolutely certain and amounts to actual knowledge of the intrinsic nature of the world. The reason is that, without God, the human mind loses its divine nature and acquires a lower ontological status, but then, the more imperfect and randomly casual its origins are, the more fallible the nature of the mind must be supposed to be, and the less reasonable it becomes to believe that subjective certainties may correspond to metaphysical truths instead of being mere anthropological features, a psycho-physical limit:

Perhaps there may be someone who would prefer to deny the existence of so powerful [i.e. absolutely omnipotent] a God rather than believe that everything is uncertain. Let us not argue with them, but grant them that everything said about God is fiction [this is the “internalist” perspective]. According to their supposition, then, I have arrived at my present state by fate or chance or a continuous chain of events, or by some other means; yet since deception and error seem to be imperfections, the less powerful they make my original cause, the more likely it is that I am so imperfect as to be deceived all the time. [AT VII, 21; CSM II, 14]

Either there is an omnipotent God, in which case my nature is as perfect as possible but no mathematical truth is immutable and hence (MDA) holds against the atheist, or there is no omnipotent God, in which case the atheist can avoid (MDA), but then human nature must be supposed imperfect and prone to err in any possible circumstances. Even without the challenge provided by (MDA), the atheist can at most have single instances of psychological reassurance, no absolute or metaphysical certainty.

²⁰ Here I agree with E. N. Curley, *Descartes against the Sceptics* (Oxford: Blackwell, 1978), pp. 29 and ff.

Conclusion: from Descartes to Gödel

Don Juan has only a limited range of possible replies to the Cartesian attack. If no resort to a metaphysical principle is allowed, perhaps mathematics can represent the foundation of the scientific encyclopedia under construction, but then mathematics itself must be provided with an epistemological justification that can prove its absolute certainty, its empirical truth, and hence explain why it fits the physical world or why the world conforms to mathematical laws. The alternatives are few.

1) Empirical externalism

For a long time, until the revision of calculus and then the foundationalist crisis, first in geometry, then in algebra and finally in set theory, Don Juan could refer to the successful application of mathematics to external reality and its remarkably accurate agreement with observations, predictions and experiments.

2) Logicism

After the foundationalist crisis, Don Juan could try to provide mathematics with a logical foundation, relying on those “empty” first principles that, as we have seen, Descartes is inclined to leave out of consideration. Unfortunately, in the *Principia Mathematica* logic itself is discovered to be ontologically demanding.

3) Constructivist intuitionism

Don Juan may accept the Cartesian method of clear and distinct ideas, and the corresponding constructive, modular methodology, and combine it with a Kantian view of internal certainty. Unfortunately, a lot of mathematics then needs to be substantially revised or abandoned, while the malicious demon hypothesis is either simply frozen in the noumena/phenomena distinction or left unanswered.

4) Formalism

Finally, Don Juan may try to see if mathematics can provide its own foundation by proving its own consistency. From a Cartesian perspective, the formalist approach, with its self-referential nature, is clearly the most interesting alternative open to the mathematical atheist. Unfortunately, we know that the very application of a self-referential procedure that makes possible the last two stages in the sceptical/dogmatist escalation, i.e. (MDA) and the *cogito*, is also what allowed Gödel to demonstrate that consistency of number theory cannot be established by the narrow logic permissible in metamathematics. Thus, from a formalist perspective, Descartes was right and Don Juan was wrong: in the absence of a metaphysical warrant, we are not justified in having absolute faith in arithmetic, since not even arithmetic can prove the impossibility of its own inconsistency and hence provide its own foundation.

Gödel's theorem may then be interpreted as the vindication of Descartes' suspicion that, without a validating metaphysical principle, not even mathematics could provide the ultimate, absolutely certain foundation of science. Commenting on the epistemological importance of Gödel's result, Hermann Weyl, himself a constructivist, once said:

God exists since mathematics is consistent, and the devil exists since we cannot prove its consistency.

This is probably the best reply Sganarelle could have offered to Don Juan's mathematical scepticism.

Appendix: Mathematical Scepticism and Eternal Truths

This appendix is not the proper place to defend a particular interpretation of Descartes' *voluntarism*—the view that God's omnipotence has absolutely no limits, so that he could make even necessary truths false and contradictions true, if he so wished. Its aim is rather that of clarifying a few central aspects of Descartes' position to help the reader to set up the problem of mathematical scepticism, as discussed in this article, against such a crucial doctrine in Descartes' philosophy.

The "Secrecy Thesis"

Unjustified claims have been made about Descartes' "secrecy" in discussion of the voluntarist doctrine at least since Bréhier's article.²¹ For reasons that seem to be largely rhetorical, Bréhier writes that

[p. 192] The Cartesian theory of the creation of the eternal truths occupies, in the body of Descartes' writings, a rather curious position. [...] [p. 193] not a word of it is breathed in the *Discourse*, nor in the *Meditations* and *Principles*; and not until eleven years later [i.e. after a series of three letters to Mersenne written in 1630] did it happen to reappear, thanks to an objection of Gassendi's (who certainly never dreamed of attributing such ideas to Descartes) in the Replies to the Fifth Set of Objections. [...] [p. 201] the theory of the creation of the eternal truths [is] a theory that Descartes wanted to make public as early as 1630, yet of which he said not a word in his books.

Not uncommonly, Bréhier blames the opacity of the message on Descartes' prudence and his desire to hide his real thoughts about the issue, disregarding the fact that Descartes, somewhat uncharacteristically, had written to Mersenne

Please do not hesitate to assert and proclaim everywhere that it is God who has laid down these laws in nature just as a king lays down laws in his kingdom.²²

Moreover, rather inconsistently, in the second half of his article Bréhier analyses Poiret's, Leibniz's and Spinoza's explicit and critical discussions of Descartes' theory of creation and remarks

[p. 204] The great Cartesians of the seventeenth century—Spinoza, Malebranche, and Leibniz—did not retain the theory of the creation of the eternal truths and in fact took it to task more or less violently.

Obviously, the theory was rather more clearly present in Descartes' writings than Bréhier is ready to acknowledge. It was described by Descartes in his letters, but what about his "printed books", those texts that would have been commonly available to his readers? As Bréhier acknowledges, Descartes discusses the voluntarist doctrine at some length in his Replies to the Objections, especially in his Reply to Gassendi, but then, surprisingly, Bréhier does not provide any justification for not accepting the Objections and Replies as an integral part of the *Meditations*. Likewise, he fails to explain why we should undervalue the philosophical importance of what Descartes is willing to write in his Replies, which is certainly one of his published texts. Descartes did not have to give a full presentation of his voluntarist doctrine in the main text of the *Meditations* if

- he took it to be a reliable and perhaps not very controversial *working* thesis that he could safely employ to develop his line of reasoning;
- it was a doctrine that was immediately connected only with one topic discussed in the

²¹ È. Bréhier, "The Creation of the Eternal Truths in Descartes' System" (1937) repr. in W. Doney (ed.), *Descartes: a Collection of Critical Essays* (London: Macmillan, 1967), pp. 192-208.

²² Letter to Mersenne, 15 April, 1630, AT I, 143-6; CSMK, III, 23.

Meditations, namely the possibility of conceiving an absolutely omnipotent malicious demon; and finally

- he had the opportunity to clarify his thoughts in the Replies.

We need to recall that, in the *Preface to the reader*, Descartes had asked his readers to be kind enough to take this part of the book seriously:

(AT VII, 10; CSM II, 8) But I certainly do not promise to satisfy my other readers straight away on all points, and I am not so presumptuous as to believe that I am capable of foreseeing all the difficulties which anyone may find. [...] Next, I will reply to the objections of various men of outstanding intellect and scholarship who had these *Meditations* sent to them for scrutiny before they went to press. [...] I therefore ask my readers not to pass judgement on the *Meditations* until they have been kind enough to read through all these objections and my replies to them.

Certainly Descartes was the first to take his *Replies* very seriously. In a letter to a writer now unknown and usually referred to as Hyperaspistes, whose objections Descartes did not manage to include in the edition of the *Meditations*, he wrote:

There is no force in what you say about the nature of a triangle [Kenny's footnote: Hyperaspistes had argued against Descartes' doctrine of the free creation of eternal truths. 'Let God do whatever he can; let us suppose *per impossibile*, that he never thought of a triangle: yet suppose you are in the world as you now are: would you not agree that it was true that the three angles of a triangle equal two right angles?']. As I have insisted in several places, when God or the infinite is in question, we must consider not what we can comprehend—for we know that they are quite beyond our comprehension—but only what conclusions we can reach by an argument that is certain. To find what kind of causal dependence these truths have on God, see my replies to the Sixth Objections, article 8.²³

So much for secrecy, one may say, but more recently Bennett²⁴ seems to have taken Bréhier's remarks at face value. He has reinforced the “secrecy thesis” by transforming Descartes' supposed reluctance into an intentional desire to hide the doctrine. In a quick, compelling but unjustified shift of perspectives we are told that:

[160-1, emphasis added] This ‘creation’ or ‘voluntarism’ doctrine *does not appear* in the *Meditations*, the *Discourse on the Method* or the *Principles of Philosophy*. Descartes first declared it in three private letters to Marin Mersenne, most of a decade before his first published work appeared. It glows luminously just behind something Descartes wrote in reply to the Second Objections to the *Meditations*, as we shall see; and he *announced it openly* in his response to (Gassendi's) Fifth Objections. [...] Furthermore, when first announcing the doctrine, he urged Mersenne to ‘assert and proclaim [it] everywhere’. His [Descartes'] reason for *keeping the doctrine out of sight* in the major published works was evidently not that he was unsure of its truth. [...] His [Descartes'] reason for *not revealing* his voluntarism doctrine in the *Meditations*, whatever it was, cannot excuse his writing things that are condemned by it.

Bennett formulates the “secrecy thesis” more ambiguously than Bréhier, but the substantial problem remains the same: we are still not told why the Objections and Replies should not count as a thoroughly explicit, intentional and public discussion of the voluntarist doctrine by Descartes himself.

The “secrecy thesis” appears then to be unjustified, but what would be the advantage of erasing the doctrine from Descartes' “published texts” in the first place? None, it seems, in Bréhier's case, where, given other inconsistencies in his article, it may well be just a matter of rhetorical presentation. Bennett's case is different, but before seeing why, we need to introduce a brief analysis of his interpretation of the voluntarist doctrine.

Voluntarism in the *Meditations*

²³ Letter to Hyperaspistes, August 1641, AT III, 430; CSMK, III, 194.

²⁴ J. Bennett, “Descartes' Theory of Modality”, *The Philosophical Review* 1979, pp. 639-667, rep. in J. Cottingham, *Descartes* (Oxford: Oxford University Press, 1998), pp. 160-185.

Scholars agree that Descartes holds that

V) necessary truths are made true by God's voluntary act

but disagree about a number of interpretative and philosophical questions raised by (V), especially, in this context, how Descartes thinks (V) can be coherently implemented, whether *ontically*

O) God has created and can re-engineer the essential nature of the universe;
or *conceptually*,

C) God has created and can reformat the conceptual limits of the human mind;
without running into a modal collapse:

MC) there is only one modality and necessary truths are not necessary after all, but merely contingent.

Here is (V) in Descartes' words:

I turn to the difficulty of conceiving how God would have been acting freely and indifferently if he had made it false that the three angles of a triangle were equal to two right angles, or in general that contradictories could not be true together. It is easy to dispel this difficulty by considering that the power of God cannot have any limits, and that our mind is finite and so created as to be able to conceive as possible the things which God has wished to be in fact possible, but not to be able to conceive as possible things which God could have made possible, but which he has nevertheless wished to make impossible. The first consideration shows us that God cannot have been determined to make it true that contradictories cannot be true together, and therefore that he could have done the opposite. The second consideration shows us that even if this be true, we should not try to comprehend it, since our nature is incapable of doing so. And even if God has willed that some truths should be necessary, this does not mean that he willed them necessarily; for it is one thing to will that they be necessary, and quite another to will this necessarily, or to be necessitated to will it. I agree that there are contractions which are so evident, that we cannot put them before our minds without judging them entirely impossible [...]. But if we would know the immensity of his power we should not put these thoughts before our minds [...].²⁵

Let us now return to the question left unanswered at the end of the previous section. The alleged absence of the voluntarist doctrine from the *Meditations* helps to support the view that there is no substantial need for it in any of the various arguments developed in the *Meditations* themselves. If the *Meditations* can do without (V)—as Bennett explicitly argues, see below—obviously there is no need for (O), which can be more easily replaced by (C), whose evaluation, in terms of textual and philosophical adequacy, may then disregard the kind of theoretical work that (V) is actually supposed to be doing in the text.

Bennett refers to Descartes' *use* of (V),²⁶ but writes that:

[p. 184] Voluntarism casts no useful light on those aspects of the *Meditations* that have received most attention: the truth rule, divine veracity, the relation between those, the Cartesian Circle.

Now, insofar as this statement is correct, it is inconsistent with the "secrecy thesis": why should Descartes have given a prominent role to (V) if it was only a marginal component in his system? And insofar as the statement is partly incorrect, it is so in a very significant sense: we have seen that Descartes does employ (V), but in connection with one of "those aspects of the *Meditations* that have received most attention" that Bennett himself does not mention, namely the discussion of the hyperbolic doubt and the formulation of (MDA). Descartes himself draws out the connection between (V) and the problem of mathematical scepticism:

As for the eternal truths, I say once more that they are true or possible only because God knows them as true or possible. They are not known as true by God in any way which would imply that they are true independently of him. [...] Those who have no higher thoughts than these [i.e. merely linguistic knowledge of the word "God"] can easily become atheists; and because they perfectly

²⁵ Letter to Mesland, 2 May 1644, AT IV, 118-9; CSMK, III, 235.

²⁶ Cf. Bennett, *art. cit.*, footnote 24 on p. 172, and p. 166.

comprehend mathematical truths and do not perfectly comprehend the truth of God's existence, it is no wonder they do not think the former depend on the latter. But they should rather take the opposite view, that since God is a cause whose power surpasses the bounds of human understanding, and since the necessity of these truths does not exceed our knowledge, these truths are therefore something less than, and subject to, the incomprehensible power of God.²⁷

Thus, the relevance of (V) for the correct understanding of (MDA) and of mathematical scepticism has been convincingly defended by Bréhier, who shares (O):

[p. 200] But in the *Meditations* we find that the theory of the creation of eternal truths, though nowhere explicitly formulated, is presupposed in two essential steps of Descartes' thinking; viz., where we are able to carry doubt much further than it has ever gone before and where we are able to free ourselves of doubt with much greater assurance than ever before. [...] Hence, to play his part, the evil genius—that imaginary being on whom Descartes has conferred the omnipotence of God without His goodness—must be the creator of essences, a creator who is completely arbitrary and continually changing. [...] [p. 201] We now see that the celebrated *loci classici* of hyperbolic doubt and the divine guarantee in the *Meditations* cannot be explained unambiguously without the theory of the creation of the eternal truths.

So perhaps a better way to describe the place of the voluntarist doctrine in the *Meditations* would be to say that it is *presupposed* as a working tool in the main text, and hence *explicitly discussed* in the Replies.

The Ontic Interpretation

Broadly speaking, the interpretation of mathematical scepticism provided in this paper agrees with Curley's "ontic" interpretation of Descartes' voluntarism (probably the most satisfactory provided so far)²⁸ and with his criticisms of Wilson's and Frankfurt's works²⁹ on the same topic:

So the suggestion is that we should understand Descartes' doctrine of the creation of the eternal truths as involving, not a denial that there are necessary truths, but a denial that those which are necessary are necessarily necessary. To think of these truths as created is neither to think that they are not necessary, nor to think that there was a time when they were not necessary, but to think that it is not necessary that they be necessary. Iterated modalities in the timeless present express Descartes' thought better than his own temporal language does. (p. 581)

It is remarkable that the passage in the *Fifth Meditation* (AT VII, 67; CSM II, 46), where Descartes relies on the concept of necessity to qualify and prove God's existence, is the only passage, in the original Latin version of the six *Meditations*, in which Descartes uses the concept of necessity at all. What we usually call "necessary truths" are better understood as eternal truths created *with* the universe and hence *with* time by a necessarily existent God, the only entity that, strictly speaking, can be said to enjoy a necessary status. This is not just a matter of vocabulary. In discussing the ontological status of mathematical or logical truths, Descartes appears to privilege an analysis in terms of a *de re* temporal logic—P is necessarily true if and only if, for any time t, it is never the case that P is false at t—instead of a modal logic—P is necessarily true if and only if there is no possible world in which P is false.³⁰ And

²⁷ Letter to Mersenne, 6 May, 1630 AT I, 149-150; CSMK, III, 24-25.

²⁸ E. M. Curley, "Descartes on the Creation of the Eternal Truths", *The Philosophical Review* (93.4), 1984, pp. 569-597.

²⁹ H. Frankfurt, "Descartes on the Creation of the Eternal Truths", *The Philosophical Review* (86.1), 1977, pp. 36-57; M. Wilson, *Descartes* (London: Routledge, 1977, rep. with corrections 1982).

³⁰ This can help to explain why in Descartes the "arrow of time" appears to necessitate, i.e. can transform a contingent fact into a necessary fact, as in the Third Meditation, where Descartes equates the modal value of a historical event with the modal value of a mathematically true proposition: "(AT 36; CSM II,

despite Curley's complaint, the former can be interpreted as consistent with a reading of (V) that does not necessarily entail (MC) any more easily than the latter:

The mathematical truths which you call eternal have been laid down by God and depend on him entirely no less than the rest of his creatures. [...] it is God who has laid down these laws in nature just as a king lays down laws in his kingdom. [...] It will be said that if God had established these truths he could change them as a king changes his laws. To this the answer is: 'Yes he can, if his will can change.' 'But I understand them to be eternal and unchangeable.'—'I make the same judgement about God.' 'But his will is free.'—'Yes, but his power is beyond our grasp.' In general we can assert that God can do everything that is within our grasp but not that he cannot do what is beyond our grasp.³¹

In Descartes, eternal truths are still "things" or essences freely *established* by God, who has, but does not exercise, the power to modify them, thus ensuring their permanence and immutability. "Established by God" does not entail "contingent":

For it is certain that he [God] is the author of the essence of created things no less than of their existence; and this essence is nothing other than the eternal truths. I do not conceive them as emanating from God like rays from the sun; but I know that God is the author of everything and that these truths are something and consequently that he is their author. [...] he was free to make it not true that all the radii of the circle are equal—just as free as he was not to create the world. And it is certain that these truths are no more necessarily attached to his essence than are other created things. [...] from all eternity he willed and understood them to be, and by that very fact he created them. Or, if you reserve the word created for the existence of things, then he established them and made them.³²

According to (O), the "necessity" Descartes has in mind is a *de facto* not a conceptualist necessity: "necessarily P" means that "with the significant, yet disregardable exception of God's omnipotence, no matter what happens, it is never the case that $\neg P$ " because God did not, does not and will not wish $\neg P$ to be the case, not because God could not make, or have made, $\neg P$ true. God could have wished $\neg P$ to be the case, but then we would no longer perceive P as necessary; at the same time, there is no risk of any modal shift, since God could but will not "change his mind" about the necessary status of P, as this would imply a form of imperfection, viz. either maliciousness or lack of power. It follows that, in Descartes, (O) does not entail the sceptical position implicit in (MC) but rather:

G necessary truths are necessary in the sense that, although only God could change them, it is certain that he will not wish to change them, because he is perfect, benevolent and truthful. If (MC) were the case, this would mean the end of any possible certainty, yet (MC) can be the case only if one concentrates just on the omnipotence thesis, disregarding the fundamental fact that the latter is strictly intertwined with the other three properties listed in (G): (O) would entail (MC), instead of the more qualified and specific (G), only if God were less perfect, benevolent or truthful than he is, e.g. only if he were a malicious demon.

25) [...] let whoever can do so deceive me, he will never bring it about that I am nothing, so long as I continue to think I am something; or make it true at some future time that I have never existed, since it is now true that I exist; or bring it about that two and three added together are more or less than five, or anything of this kind in which I see a manifest contradiction".

³¹ Letter to Mersenne, 15 April, 1630, AT I, 145-6; CSMK, III, 23.

³² Letter to Mersenne, 27 May, 1630 AT I, 152-3; CSMK, III, 25.

The Conceptualist Interpretation

We have seen that the main alternative to (O) is (C). (C) appears to favour the view that Descartes' philosophy can best be understood if univocally interpreted as a thoroughly subjectivist theory. Descartes' philosophical style and vocabulary certainly allow a similar interpretation, and yet, reading Descartes from Hume's or even Kant's perspective, may be very inadequate.

Bennett has provided the most satisfactory version of (C):

[p. 167] Descartes had a subjectivist theory about the nature of necessity [that is] [p. 166] God created modal truths by making us unable to conceive of impossibilities [or, to put it in other words] [p. 168] Given that all modal truths are at bottom truths about what we can conceive, and given that God made us how we are (this being a truism for Descartes), it follows that God gives modal truths their status as truths. [p. 181] According to voluntarism as I understand it, for P to be necessary is for P to relate thus and so to our intellectual limits; and those limits are innate, in that God gave them to us as part of our human birthright.

It may be worth illustrating the difference between (O) and (C) by discussing an example. According to Bennett:

Descartes juxtaposes

(*) a thesis relating necessary truths to God, with

(**) a thesis relating them to us.

If we illustrate (*) with *It is not impossible for God to make an uphill without a downhill* [this is our (V)], then (**) is illustrated by *We cannot conceive of an uphill without a downhill*. [...] If we construe (*) as entailing that there could be an uphill without a downhill because God could create one [this is our (O)], the companion thesis would be (**) that we (wrongly) think there could not be an uphill without a downhill because we cannot conceive it. [p. 165, my typographic modification]

Note how Bennett assumes, without further justification, that (O) entails (MC), yet we have seen that this is not necessarily the case. Descartes subscribes only to the more limited thesis (G), so the second part of the quotation does not follow, and the modal collapse can be avoided without necessarily accepting (C). In Descartes, we should construe (*) as entailing that, if God had wished, there could be an uphill without a downhill because God could have created one, so that the companion thesis is (**) that, since God has wished, and keeps on wishing, that whenever there is an uphill there is also a downhill we (rightly) think there could not be an uphill without a downhill because we cannot conceive it, although we can conceive God causing things to be otherwise. Once we know how things are and God's nature, we also know that modal statements are perfectly safe.

Correctly interpreted as entailing (G) rather than (MC), (O) turns out to be the source of no global problem, but only of a specific issue in Descartes. On the one hand, once the existence of a benevolent, truthful and perfect God has been proved, all logical arguments can work in exactly the same way as we would expect them to work, or at least no worse than they would work if we were to accept (C) anyway. On the other hand, the ontic interpretation of the stability of modalities obviously leads to the specific problem represented by the Cartesian Circle: it is not clear how one could rely on the use of modalities to prove the existence of the entity that guarantees their reliability. This, however, is not an argument against the interpretation of Descartes' position as O-like. On the contrary, it shows that precisely because Descartes probably viewed (V) in terms of (O) + (G), we can speak of the Cartesian Circle as an actual issue in his philosophy.

Though not necessarily inconsistent with the reconstruction of mathematical scepticism given in this article, (C) turns out to be more problematic for other reasons. It is weaker and philosophically more plausible than (O)—this explains why it can be shown to be *compatible*

with most, though by no means all,³³ passages where Descartes *discusses*, more or less explicitly, his voluntarist doctrine—but it can be argued that it was not Descartes’ own view, for either (C) itself or its consequences fail to harmonise completely with other parts of his philosophy.

Most notably, Descartes seems to require and indeed employ an ontic rather than a merely conceptualist understanding of modal notions in order to describe God’s existence and to construct the ontological proof. As regards the latter point, if (C) is the case, it becomes difficult to make sense of the argument, which requires a *de re* modal escalation, not a merely conceptualist one. As regards the former point, Bennett refers to Descartes’ discussion of the concept of possibility (AT VII, 150 ff.; CSM II, 107) to support (C), but while this passage is also perfectly understandable in terms of (O), there is a much more enlightening text, about God’s necessary existence, which poses a remarkable problem for any conceptualist interpretation (the “objectivist” nature of God’s necessary existence is even more unmistakable in the original Latin text):

It is not that my thought makes it so, or imposes any necessity on any thing; on the contrary, it is the necessity of the thing itself, namely the existence of God, which determines my thinking in this respect. For I am not free [more literally: it is not up to me] to think of God without existence (that is, a supremely perfect being without a supreme perfection) as I am free to imagine a horse with or without wings.³⁴

It is hard to reconcile (C) with Descartes’ view that, in the case of God’s existence, it is not up to the human mind, nor is it a matter of mental framework, to establish what is or is not necessary, but only to acknowledge it. The issue is obviously connected with the possibility of constructing the ontological argument. Bennett addresses the whole problem only partly. Neither of the two objections against (C) that Bennett considers in Section V (pp. 168-170) based on God’s necessary existence really address the point. The actual objection is that Descartes himself (not the reader) believes that God exists necessarily not in the weaker sense that “we cannot conceive that God should not exist”, but in the stronger sense that “it is

³³ See Descartes’ letter to Mesland, also quoted by Bennett: “The power of God cannot have any limits. [...] God cannot have been determined to make it true that contradictories cannot be true together, and therefore that he could have done the opposite. (AT IV 118; CSMK, III, 235)”. The letter is not the only case in which Descartes makes clear his non-conceptualist interpretation of modalities, cf. for example his Letter to More, February 5, 1649: “True, our mind is not the measure of reality or of truth; but certainly it should be the measure of what we assert or deny.” (AT V, 274; CSMK, III, 364). Bennett fairly acknowledges that the letter to Mesland undermines the credibility of (C), but appears to be unwilling to draw the necessary consequence that (C) is therefore incorrect. He resists this conclusion both by trying to transform the question about the textual adequacy of (C) into a matter of degree—“My interpretation fits *nearly everything* in the voluntarist texts: this late in the day nobody will discover that the texts consistently express a single view; nothing can clear them of the charge of wavering sometimes (p. 162)” —and by arguing that other interpretations are inadequate (see pp. 175-179). In the former case, one must simply resist Bennett’s suggestion: there is a limit to hermeneutic flexibility, and the price for not respecting it is to make any interpretation plausible, even those that clearly and distinctly conflict with the relevant texts. In the latter case, arguing in favour of (C) by showing how other interpretations are also confronted by their own problems is obviously ineffective, especially when the most obvious candidate, namely (O), is still perfectly available.

³⁴ AT VII, 67; CSM II, 46. It is worth quoting the Latin text as well: “Imo sophisma hic latet; neque enim, ex eo quod non possim cogitare montem nisi cum valle, sequitur alicubi montem et vallem existere, sed tantum montem et vallem, sive existant, sive non existant, a se mutuo seiungi non posse. Atqui ex eo quod non possim cogitare Deum nisi existentem, sequitur existentiam a Deo esse inseparabilem, ac proinde illum revera existere; non quod mea cogitatio hoc efficiat, sive aliquam necessitatem ulli rei imponat, sed contra quia ipsius rei, nempe existentiae Dei, necessitas me determinat ad hoc cogitandum: neque enim mihi liberum est Deum absque existentia (hoc est ens summe perfectum absque summa perfectione) cogitare, ut liberum est equum vel cum alis vel sine alis imaginari.”

absolutely necessary that God exists”, and the conceptualist analysis of modalities can only support the former view, not the latter. So when Bennett writes:

[p. 169] You may want to object: ‘You have extricated Descartes from the tangle of bootstraps by weakening his theology from the momentous and cosmic proposition (a) that it is absolutely necessary that God exists, to the humdrum and local truth (b) that we cannot conceive that God should not exist’. That objection implies that (b) is weaker than (a), which amounts to rejecting the conceptualist analysis of modality. Even if you are right to reject it, however, my hypothesis that Descartes accepted it is all I need to abolish the bootstrap problem. If Descartes believed the conceptualist analysis of modality, and asserted voluntarism on that basis, he was not threatened with any paradox in relating the latter thesis to the thesis that necessarily God exists. Q.E.D.

the problem is that we are not yet allowed to *presuppose* that Descartes believes (C), but must prove this against possible objections. Now the objection is that Descartes holds (a), and since (b) is indeed weaker than (a), and holding both (a) and that (b) is weaker than (a) does amount to rejecting (C), as Bennett himself admits, then (C) must be rejected.

The Interpretation of Voluntarism and Mathematical Scepticism

Both (O) and (C) are confronted with the problem of explaining precisely *how* the malicious demon could make a mathematical truth doubtful. In both cases, the malicious demon must be considered to be so powerful as to be able to affect our perception of eternal truths. The ontic interpretation, defending the view that eternal truths are adequately grasped by, but independent of, the human mind, can allow the malicious demon to mislead the mind without actually modifying its nature. The Cartesian argument could be phrased along the following lines:

- 1 P is necessarily the case if and only if God has established that $\neg P$ is never the case, forever; however, as long as we do not have a divine guarantee, the fact that $\neg P$ is never the case cannot be trusted, as a criterion of absolute certainty of P, because an omnipotent malicious demon could *somehow* mislead me into believing that P is necessarily the case while $\neg P$ may still turn out to be the case, even if I cannot conceive how the “somehow” could be made any more specific.

On the other hand, the conceptualist interpretation requires the malicious demon to interact directly with the human mind, formatting it or reformatting it in such a way that it will go wrong even when eternal truths are in question. In this case, the Cartesian argument could be phrased along the following lines:

- 2 P is necessarily the case if and only if God has established to make it mentally inconceivable that $\neg P$; however, as long as we do not have a divine guarantee, the inconceivability of $\neg P$ cannot be trusted, as a criterion of absolute certainty of P, because an omnipotent malicious demon could have given me the wrong modal frame within which I interpret the modal status of P. The fact that I cannot conceive how $\neg P$ could be made true can only make explicit my mental limits, not make P any more reliable.

Neither of the two alternatives are philosophically very convincing, but from a Cartesian perspective (1) seems to be more acceptable than (2). A defender of (1) is obviously unable to explain *how* the existence of a malicious demon may make a mathematical truth, such as the fact that the three angles of a triangle are equal to two right angles, doubtful. We are only told that, if there is a malicious demon as omnipotent as God and if omnipotence is understood, ontically, as the power to do absolutely anything, then a possible (“metaphysical” in Cartesian terminology) doubt can be cast even on the certainty of eternal truths. Note that the vague nature of this “metaphysical doubt” is already sufficient for the Cartesian project. We do not need to know how P may not be the case to start entertaining the suspicion that perhaps P is not the case. Descartes himself acknowledges this much when speaking about God’s omnipotence, for example in the Replies to Sixth Objections:

There is no point in asking by what means God could have brought it about from eternity that is was not true that twice four make eight, and so on; for I declare that this is unintelligible to us (AT VII, 436; CSM II, 294)

(MDA) only needs to make us a little more suspicious about the absolute certainty of the eternal truths than we would have been before contemplating the possibility of an omnipotent malicious demon.

Since (2) adopts a much less strong conception of modalities, one can expect it to be able to overcome the procedural difficulty confronting (1) more easily. God creates modalities not *in re*, by creating the universe, but *in mente*, by establishing what is in/conceivable, so (2) seems to have an easier task in explaining *how* an omnipotent malicious demon may mislead us, by giving us the wrong modal framework. In this scenario, P is actually false but we are made mentally incapable of conceiving $\neg P$ as logically possible.

There are at least two problems with (2), but let us first clear the ground of two issues *not* at stake here: first, (2) appears philosophically more plausible than (1), but what is in question is whether it is also Descartes' position and hence consistent with the rest of his philosophy; and secondly, (2) should not be confused with a form of sceptical psychologism. The fact that, for example, we are unable to conceive an actual space in which the fifth postulate of Euclidean geometry does not hold has nothing to do with the inconceivability discussed in (2). After all, we are able to prove that non-Euclidean geometries are logically possible, i.e. conceivable in terms of (C). (2) suggests something more radical. Let us turn to the problems.

The first problem with (2) concerns the modal status of what we fail to know. As one may reasonably expect, Descartes holds a correspondence theory of truth:³⁵ if "P" is true this is because P is the case and "P" describes P as being the case, and if "P" is false then this is because $\neg P$ is the case yet "P" describes P as being the case. What is the modal status of $\neg P$ according to (MDA) when "P" is interpreted as an eternal truth in the sense specified by (C)? If the question is not completely meaningful, then we may prefer to opt for (1), according to which the question would be perfectly plausible: God creates the world in which $\neg P$ is the case, but according to (MDA) we do not know whether he can make us believe, wrongly, that P is necessarily true. If the question is meaningful, then, whatever the answer is going to be, it seems that the modal status of $\neg P$ cannot be mind-dependent, so (C) is incorrect after all.

The first problem caused by (MDA) about the acceptability of (C) concerns the apparent impossibility of being wrong when modalities interpreted in conceptualist terms are in question. This leads us to the second problem, concerning the role played by the malicious demon. In the First Meditation, Descartes argues that God, not a malicious demon, is the creator of the human mind:

[AT VII, 21; CSM II, 14] And yet, firmly rooted in my mind is the long-standing opinion that there is an omnipotent God who made me the kind of creature that I am.

but that there may be, apparently at the same time, an omnipotent malicious demon who is misleading us

[AT VII, 22; CSM II, 15] I will suppose therefore that not God, who is supremely good and the source of truth, but rather some malicious demon of the utmost power and cunning has employed all his energies in order to deceive me.

In the Third Meditation, however, Descartes speaks only of whether there is a God and whether he may be a deceiver. If there is a God and he is a deceiver, we could be "Children of a Lesser God", as it were, creatures of a malicious demon who plays even with our most fundamental certainties. The text of the Third Meditation is compatible both with (1) and (2), but is Descartes considering the possibility that there may be a God *and* a malicious demon, or only

³⁵ Curley, *art. cit.*, p. 572.

that there may be a God who is malicious? If the hypothesis that there may be a malicious demon is treated as being different from the suggestion that (a) God could be a deceiver and (b) the malicious demon may be our creator, as seems to be the case in the First Meditation, then this makes it hard to explain how the malicious demon could “undo” God’s work so substantially, i.e. provide us with such minds that we go wrong even when considering eternal truths. All this does not represent a problem for (O) + (1), because in that case the malicious demon would be misleading us epistemically—he makes us perceive eternal truths wrongly—not ontically, i.e. we are provided with the wrong minds.

While (C) + (2) is not incompatible with (MDA), the impression we had before, namely that (2) could explain the *how* question better than (1), now appears less justified. We can now summarise the difficulty thus: the more ontically (V) is interpreted the less theoretically plausible it becomes, but also the more epistemically (MDA) can be interpreted, and hence the more theoretically plausible it can become; conversely, the more epistemically, conceptually or mentally (V) is interpreted, the more theoretically plausible it becomes, but the more ontically (MDA) must be interpreted. As I argued above, an ontic interpretation of (V) and an epistemic interpretation of (MDA) seem to capture Descartes’ position better.